Homework Assignment 03

Question 1 (2 points each unless noted otherwise)

1. A 9-V dc power supply generates 10 W in a resistor. What peak-to-peak amplitude should an ac source have to generate the same power in the resistor?
   
   (a) 12.73 V  
   (b) 25.5 V  
   (c) 18 V  
   (d) 12.73 V

   **Answer:** The ac source’s effective (or rms) value should also be 9 V. This means the peak value should be $9\sqrt{2}$ V, so the peak-to-peak value should be $18\sqrt{2} = 25.5$ V, so the answer is (b).

2. In the circuit, $V_1 = 1.5$ V, $V_2 = 2.5$ V, and all the resistors have value 20K. The output voltage is

   (a) $-4.5$ V  
   (b) $+4.5$ V  
   (c) $-4$ V  
   (d) $-0.5$ V  
   (e) $+0.5$ V

   **Answer:** This is a summing inverter with $V_{out} = -V_1(R_2/R_1) - V_2(R_2/R_1)$. Since all the resistors have same value, $V_{out} = -V_1 - V_2 = -4$ V, so (c) is the answer.

3. What is the 3-dB bandwidth of the circuit below?

   (a) $\approx 8$ kHz  
   (b) 31.83 kHz  
   (c) 15.92 kHz  
   (d) 100 kHz

   **Answer:** The capacitor sees an equivalent resistance $R = R_2 = 10K$ (the current source has infinite internal resistance) and the time-constant is $\tau = RC = 10 \mu$s, so that the bandwidth is $1/(2\pi\tau) = 15.92$ kHz, and (c) is the answer.
4. What is the 3-dB bandwidth of the amplifier shown below if $r_\pi = 2.5K$, $r_o = 100K$, $g_m = 40$ mS, and $C_L = C_F = 1$ nF?

(a) 795.8 Hz
(b) 1.59 kHz
(c) 5 kHz
(d) 4.71 kHz

**Answer:** If one turns off $V_I$, it is shorted to ground and $C_L$ is in parallel with $C_F$ so the effective capacitance is 2 nF. This capacitor sees an equivalent resistance $r_o = 100K$. (If one turns off $V_I$, $g_mv_\pi = 0$, and the current source is effectively removed from the circuit.) The time-constant is $\tau = RC = (2 \text{ nF})(100\text{K}) = 200 \mu$s. The bandwidth is $1/(2\pi\tau) = 795.8$ Hz, so the answer is (a).

5. The voltage gain of the amplifier shown is

(e) $\approx -5.7$
(f) $\approx 5.7$
(g) $\approx 6.77$
(h) $\approx 13.4$

**Answer:** $A_v = -R_f/R_1 = 68\text{K}/12\text{K} = 5.7$, so the answer is (a)

6. The output of the circuit shown is

(a) Sine wave with frequency $\omega$ rad/s
(b) Square wave with frequency $\omega/2\pi$ Hz
(c) Triangular wave with frequency $\omega$ rad/s
(d) Need additional information

**Answer:** With no feedback, the circuit is highly non-linear and operates as a comparator, comparing the input amplitude against 0 V. The output is a square wave for frequency $\omega/2\pi$ Hz, so the answer is (b)
7. In the circuit shown, the output voltage is
   (a) $5(1 + 8/2) = 25 \text{ V}$
   (b) $5(8/2) = 20 \text{ V}$
   (c) $\approx 15 \text{ V}$
   (d) $\approx -15 \text{ V}$
   (e) $(8/2) = 120 \text{ V}$

   **Answer:** This is a non-inverting amplifier with gain $(1 + 8/2) = 5$, so with a 5-V input the output should be 25 V. However, the op-amp is powered by a +15-V power supply, so that the output will be clamped to a value close to +15 V, so the answer is (c).

8. An engineer designs a MOSFET-based class-AB amplifier to deliver 6.25 W (sinusoidal) signal power to a 4 Ω resistive load. What is the required peak-to-peak voltage swing across the load?

   **Answer:**
   
   $P = \frac{V_{\text{rms}}^2}{R}$, so that $V_{\text{rms}} = 5 \text{ V}$, so that $V_{\text{pp}} = 14.14 \text{ V}$

9. An engineer designs a MOSFET-based class-AB amplifier to deliver 6.25 W (sinusoidal) signal power to a 4 Ω resistive load. What is the required peak-to-peak voltage swing across the load?

   (a) 9.77 V
   (b) 19.53 V
   (c) 10 V
   (d) 14.14 V
   (e) 7.07 V

   **Answer:**
   
   $P = \frac{V_{\text{rms}}^2}{R}$, so that $V_{\text{rms}} = 5 \text{ V}$, so that $V_{\text{pp}} = 14.14 \text{ V}$, so (d).

10. Consider a linear power supply consisting of a transformer, a full-wave, 4-diode bridge rectifier, smoothing capacitor, and a load current 1.2 A. By what percentage will the ripple voltage increase if the load current increases to 1.5 A?

    a) 100 %  
    b) 25%  
    c) Stay the same  
    d) 50%

    **Answer:** 25%

11. Consider a linear power supply consisting of a transformer, a full-wave, a bridge rectifier and a smoothing capacitor. Increasing the smoothing capacitor by 50% will

    (a) Reduce ripple voltage by 50% and increase maximum inrush current by 50%
    (b) Reduce both ripple voltage and maximum inrush current by 50%
    (c) Reduce ripple voltage by 50% and leave maximum inrush current unaffected
    (d) Reduce ripple voltage by 50% and increase maximum inrush current by 100%

    **Answer:** (a)
12. Consider a linear power supply consisting of a transformer, a half-wave, 1-diode rectifier, and a smoothing capacitor. The rectifier diode is now replaced with a bridge (4-diode) rectifier. Neglecting the diodes’ turn-on voltages, the ripple voltage will:

a) Decrease by a factor 4
b) Decrease by a factor 2
c) Stay the same
d) Increase by a factor 4

Answer: (b)

13. In an ac series RC circuit, if 20 VAC is measured across the resistor and 40 VAC is measured across the capacitor, the magnitude of the applied voltage is:

(a) \( \approx 60 \text{ VAC} \)  (b) \( \approx 55 \text{ VAC} \)  (c) \( \approx 50 \text{ VAC} \)  (d) \( \approx 45 \text{ VAC} \)

Answer: The applied voltage is \( V_{IN} = V_R + jV_C \), so that \( |V_{IN}| = \sqrt{V_R^2 + V_C^2} = \sqrt{2000} \approx 45 \text{ VAC} \). Thus (d) is the answer.

14. What is the magnitude of the current phase angle for a 5.6 \( \mu \text{F} \) capacitor and a 50-\( \Omega \) resistor in series with a 1.1 kHz, 5 VAC source?

(a) 72.9°  (b) 62.7°  (c) 27.3°  (d) 17.1

Answer: The impedance of the RC circuit is \( R - 1/j2\pi fC = 50 - j25.84 \Omega \). The magnitude of the phase angle is \( |\tan^{-1}(-25.84/50)| = 27.3^\circ \). Thus, (c) is the answer.

15. Estimate the current through the red LED in the circuit shown.

(a) 0 mA  
(b) 16 mA  
(c) 18 mA  
(d) 13 mA

Answer: \( V_f \) for a Si diode is nominally 0.7 V, but LEDs are made from other semiconductor material and LEDs’ turn-on voltages are different. In particular, red LEDs have \( V_f \approx 1.6 \text{ V} \). Thus, the current in the circuit is \( I = (6 - 1.6)/330 \approx 13 \text{ mA} \), and the answer is (d).
16. A diode for which you change the reverse bias voltage to change the junction capacitance is called _________ diode and these are used in tuned circuits.
   
   (a) varactor (b) tunnel (c) switching (d) Zener

**Answer:** Option (a)

17. A filtered full-wave rectifier voltage has a smaller ripple than does a half-wave rectifier voltage for the same load resistance and capacitor values because:

   (a) There is a shorter time between peaks  
   (b) There is a longer time between peaks  
   (c) The larger the ripple, the better the filtering action  
   (d) None of the above

**Answer:** Option (a).

18. The PIV across a nonconducting diode in a bridge rectifier equals approximately:

   (a) half the peak  
   (b) 2 × the peak  
   (c) the peak  
   (d) 4 × the peak

value of the transformer secondary voltage.

**Answer:** Option (c)

19. What is the current through the ideal diode?

   (a) 1 mA  
   (b) 0.975 mA  
   (c) 0.942 mA  
   (d) 0.867 mA

**Answer:** For an ideal diode there is no forward voltage drop, so \( I = \frac{12}{12k \Omega} = 1 \text{ mA} \), so option (a) is the answer.

20. With a 12-V supply, a silicon diode, and a 370 \( \Omega \) resistor in series, what voltage will be dropped across the diode?

   (a) \( \approx 0.3 \text{ V} \)  
   (b) \( \approx 0.7 \text{ V} \)  
   (c) \( \approx 0.9 \text{ V} \)  
   (d) (a) \( \approx 1.4 \text{ V} \)

**Answer:** Option (b)
Question 2 For the circuit shown, $V_{PS} = 5 \text{ V}$, $R = 5\text{K}$, $v_y = 0.6 \text{ V}$. The input is $v_i = 0.1 \sin(\omega t) \text{ V}$.

Draw a small-signal ac model and provide numerical values for the model parameters. Then determine and expression for the time-varying component of the output voltage $v_O$.

Provide your answer to 3 significant digits. (6 points)

Solution

The dc current through the diode is $I_{DQ} = (V_{PS} - v_y)/R = (5 - 0.6)/(5\text{K}) = 0.88 \text{ mA}$. The small-signal (incremental) resistance of the diode is $r_d = V_T/I_{DQ} = (26 \text{ mV})/(0.88 \text{ mA}) \approx 30 \Omega$. A small-signal ac model is shown below.

The small-signal output voltage is

$$v_O(t) = \frac{R}{R + r_d} v_i(t) = \frac{5K}{5K + 30 \Omega} (0.1) \sin(\omega t) = 0.0994 \sin(\omega t) \text{ V}$$
Question 3  An engineer measures the bandwidth of the circuit below by driving it with a sinusoidal signal and measuring the attenuation at various frequencies. She uses a scope with an input impedance of 1 MΩ with a × 1 probe, and then a × 10 probe. Complete the following table (6 points)

<table>
<thead>
<tr>
<th>True BW in Hz</th>
<th>Measured BW in Hz (× 1) probe</th>
<th>Measured BW in Hz (× 10) probe</th>
</tr>
</thead>
<tbody>
<tr>
<td>175 Hz</td>
<td>334 Hz</td>
<td>191 Hz</td>
</tr>
</tbody>
</table>

Solution

This is a simple 1th-order system with bandwidth $B = 1/(2\pi RC)$. Here $R$ is the equivalent resistance that the capacitor $C$ sees.

True bandwidth calculation: the Thevenin equivalent resistance that $C$ sees is simply $R_1$, so

$$B_{\text{True}} = \frac{1}{2\pi RC} = \frac{1}{2\pi (910 \times 10^3)(1 \times 10^{-9})} = 175 \text{ Hz}$$

× 1 Probe bandwidth calculation: the probe+scope has a 1M resistance that is effectively in parallel with $R_1$, so that the Thevenin equivalent resistance that $C$ sees is $1M\|R_1$, so that

$$B_{\times1 \text{ probe}} = \frac{1}{2\pi (R\|R_{\text{scope}})C} = \frac{1}{2\pi (910 \times 10^3)|| (1 \times 10^6)(1 \times 10^{-9})} = 334 \text{ Hz}$$

× 10 Probe bandwidth calculation: ×10 probes increase the 1 MΩ probe+scope resistance to 10 MΩ, so that the Thevenin equivalent resistance that $C$ sees is $10M\|R_1$, so that

$$B_{\times10 \text{ probe}} = \frac{1}{2\pi (R\|R_{\text{scope}})C} = \frac{1}{2\pi (910 \times 10^3)|| (10 \times 10^6)(1 \times 10^{-9})} = 191 \text{ Hz}$$
**Question 4** What is $v_o$ in the following circuit if $v_{REF} = 1.2 \, V$, $R_1 = 680 \, \Omega$, and $R_2 = 200 \, \Omega$? (4 points)

![Circuit Diagram]

**Solution** The negative feedback loop forces the node at the junction of $R_1$ and $R_2$ to be equal to the reference voltage. The current through $R_2$ is $1.2/200 = 6 \, mA$, which also flows through $R_1$. Thus, the output voltage is $1.2 + 0.006 \times 680 = 5.28 \, V$

**Question 5** What is the voltage across a capacitor after being charged from a 100 V source for a period of one time constant? The initial voltage across the capacitor is 0 V. (4 points)

**Solution** The voltage across the capacitor is $v_c(t) = 100(1 - e^{-t/\tau})$. Thus, $v_c(\tau) = 100(1 - 1 - e^{-1}) = 63.3 \, V$. Thus, (c) is the answer. Note, it is a standard result that one should know by heart—a capacitor charges to $\approx 63\%$ of its final value after one time constant.

**Question 6** Determine absolute value of the peak current through the load resistor? Assume $V_p = 0.7 \, V$ for the diodes. (4 points)

![Diode Circuit Diagram]

**Solution** When $v_i = 10 \, V$, $D_1$ is reverse-biased and an open circuit. $D_2$ is forward biased and has a $0.7 \, V$ voltage drop across it. It is in series with $R_L$ and the left 2K resistor, so the current that flows is $I = (10 - 0.7)/(2K + R_L) = 9.3/(4K) = 2.235 \, mA$. When $v_i = -10 \, V$, then $D_2$ is reverse-biased but $D_1$ is forward biased. The current that flows is again $2.235 \, mA$, but now it flows in the opposite direction. Regardless the answer is $2.235 \, mA$. 

8
**Question 7** What is the voltage gain $A_v = v_o/v_s$ of the amplifier below if $g_m = 0.04$ S and $r_o = 100K$? (5 points)

![Amplifier Circuit](image)

**Solution**

Note that $r_o$ is in parallel with the 10K resistor. KCL at the output gives

$$-g_m v_\pi + \frac{v_o}{(r_o||10K)} = 0$$

Further, from the circuit, $v_\pi = v_s$, so that

$$-g_m v_s + \frac{v_o}{(r_o||10K)} = 0$$

Solving for $A_v = v_o/v_s$ gives $A_v = -g_m(r_o||10K) = -0.04(100K||10K) = -363.6$

**Question 8** Consider the circuit below. What is the output voltage $V_{OUT}$ at the end of the 2.82 ms pulse? (5 points)

![Circuit Diagram](image)

**Solution**

On the rising edge the capacitor is uncharged and 15V appears across $R_1$. The voltage across the capacitor is $V_C = 15(1 - e^{-t/\tau})$ where $\tau = RC = 940 \mu s$ is the time constant. The voltage across $R_1$ is $15e^{-t/\tau}$.

At $t = 2.82$ ms, this is $5e^{-2.82 ms/940 \mu s} = 15e^{-3} = 0.747$ V.
**Question 9 (diodes, load line)** Consider the circuit below. Assume $V_{PS} = 3.5$ V, and $R = 180$ Ω. Also shown, are the LED’s voltage-current characteristics. Draw the circuit’s dc load line on the characteristics and find $I_D$ and $V_D$ (6 points)

**Solution.** On the voltage axis, mark the supply voltage: $3.5$ V. On the current axis, mark the maximum current that can flow through the resistor: $I = \frac{3.5}{180} = 19.4$ mA. Connect the two points to get the dc load line. The dc load line intersects the diode $V$-$I$ curve at around $I_D \approx 6$ mA and $V_D \approx 2.8$ V.
Problem 10  A full-wave, 4-diode bridge rectifier circuit with a 1 kΩ load operates from a 120-V (rms) 60-Hz household supply through a 10-to-1 step-down transformer. It uses silicon diodes that one can model to have a 0.7-V drop for any current. (a) What is the peak voltage of the rectified output? (3 points) (b) For what fraction of the time does the diode conduct? (5 points)

Solution

Part (a)  The peak voltage after the 10-to-1 step down is \( V_p = 12\sqrt{2} = 16.97 \text{ V} \) so the peak voltage of the rectified output is this, minus two diode drops, or

\[
V_{p(\text{load})} = 16.97 - 1.4 = 15.57 \text{ V}
\]

Part (b)  Since the rectified wave is periodic, we need to consider only one cycle of the rectified wave. Conduction starts when the output voltage from the transformer (i.e., \( 16.97 \sin(\omega t) \)) equals two diode drops. In other words, when \( 16.97 \sin(\omega t) = 1.4 \). Solving yields \( \theta = \omega t = 0.083 \text{ radian} \). The rectified half-cycle represents \( \pi \) radians, so conduction starts \( (0.083/\pi) \times 100 = 2.63\% \) into the half-cycle. By symmetry, conduction stops at \( 100 - 2.63 = 97.37\% \) of the half-cycle, and the diodes conduct \( 100 - 2 \times 2.63 = 94.7\% \) of the time.
Problem 11 An engineer designs a power supply that consists of a transformer, a full-wave, 4-diode bridge rectifier and a smoothing capacitor. She designed the supply to operate in the U.S. where the power line (mains) frequency and voltage is 60 Hz and 120 V respectively. The ripple voltage at full load is 20 mV. Estimate the ripple voltage when the unmodified supply is used in regions of Japan where the corresponding values are 50 Hz and 100 V respectively. Assume that the equivalent load resistance stays the same. (5 points)

Solution

The ripple voltage for a full-wave, 4-diode bridge rectifier is (see chapter 2 of 4th edition of Neaman’s text book):

\[ V_r = \frac{V_M}{2fRC} \]

Here \( V_M \) is the maximum (peak) voltage of the input sine wave, \( f \) is the frequency, \( C \) is the capacitance of the smoothing capacitor, and \( R \) is the load resistance. With new mains voltage \( V'_M = \frac{100}{120}V_M \) and new mains frequency \( f' = \frac{50}{60}f \), the new ripple voltage is

\[ V'_r = \frac{V'_M}{2f'RC} = \frac{(100/120)V_M}{2(50/60)fRC} = \left(\frac{100}{120}\right)\left(\frac{60}{50}\right)\left(\frac{V_M}{2fRC}\right) = \frac{V_M}{2fRC} = 20 \text{ mV} \]

That is, the ripple voltage will not change.
**Question 12** Consider the 5 V linear power supply below. The load current is 200 mA. The three-terminal regulator has a 50 dB ripple rejection ratio at 120 Hz. The forward voltage for the 1N4002 rectifier diodes is 1 V, and the 680 μF smoothing capacitor has an ESR of 0.75 Ω. (a) Estimate the ripple and average (dc) voltage just before the linear regulator. That is, at point A. (4 points) (b) Estimate the output ripple voltage. (3 points) (c) Estimate the worst-case inrush current through the diodes. Ignore the transformer winding resistance. (4 points) (d) Estimate the efficiency $\eta = \frac{P_L}{P_L + P_{diss}}$ of the power supply. (3 points)

![Power Supply Diagram](image)

**Solution**

**Part (a)** The ripple voltage just before the regulator is

$$V_r = I_L / (2fC) = 0.2 / (120 \times 680 \times 10^{-6}) = 2.45 \text{ V}$$

The peak voltage before the linear regulator is $\sqrt{2} \times 9 - 2V_D = 10.7 \text{ V}$, so the dc voltage is $10.7 - V_r / 2 = 9.5 \text{ V}$

**Part (b)** The regulator suppresses the ripple voltage by 50 dB, which is the same as a factor $10^{2.5}$, so that the output ripple voltage is $V_{ro} = 2.45 / 10^{2.5} = 7.75 \text{ mV}$

**Part (c)** The worst-case inrush current occurs when the smoothing capacitor is uncharged and power is applied right when the input voltage to the bridge rectifier crests. This voltage is $\sqrt{2} \times 9 = 12.72 \text{ V}$ and the current through a pair of diodes and the capacitor is

$$I_{inrush} = \frac{12.72 - 2V_D}{0.75} = \frac{12.72 - 2}{0.75} = 14.33 \text{ A}$$

**Part (c)** $P_L = I_L V_L = (0.2)(5) = 1 \text{ W}$. The power dissipated in the bridge rectifier is $2V_D I_L = (2)(1)(0.2) = 0.4 \text{ W}$. The power dissipated by the linear regulator is $(9.5 - 5)I_L = 0.98 \text{ W}$. The efficiency is then

$$\eta = \frac{1}{1 + 0.4 + 0.98} = 42\%$$
**Question 13** Determine $V_D$ for the circuit shown to within 0.01 V. This is not necessarily a Si diode so you can’t assume $V_D = 0.7$ V. Rather, use the diode equation $I_D = I_S [e^{V_D / V_T} - 1]$ and assume that $I_S = 10^{-13}$ A and $T = 300$ K. Use the bisection numerical method with the initial bracket values $V_L = 0.6$ V and $V_H = 0.65$ V. Organize you values neatly in a table. *(8 points)*

**Solution**

KVL gives $-V_{PS} + I_D R + V_D = 0$ For the diode

$$I_D = I_S [e^{V_D / V_T} - 1]$$

Eq. 1

Substituting this into the KVL equation and reorganizing gives

$$V_{PS} = R I_S [e^{V_D / V_T} - 1] + V_D$$

$$V_D = 5 - (2 \times 10^3)(1 \times 10^{-13})[e^{V_D / 0.026} - 1]$$

Eq. 2

Now try different values for $V_D$ substituting in the RHS of the equation above.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>$V_L$</th>
<th>$V_H$</th>
<th>$V_D = (V_H - V_L)/2$</th>
<th>$V_D$ which is from RHS of Equation 2</th>
<th>$V_D' - V_D$</th>
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<td>0.62500</td>
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</tbody>
</table>

% Matlab Script
R = 2e3;
Is = 1e-13;
VT = 0.026;
VH = 0.65;
VL = 0.6;
for i = 1:100
    VD = (VH+VL)/2;
    VDP = 5 - Is*R*(exp(VD/VT)-1);
    err = VDP-VD;
    s = sprintf('%d, %7.5f, %7.5f, %7.5f, %7.5f, %7.5f, %7.5f, %7.5f, %7.5f',i,VL,VH,VD,VDP,err);
    disp(s)
    if (err < 0)
        VH = VD;
    else
        VL = VD;
    end
    if (abs(err) < 0.01)
        break;
    end
end