Sinusoidal Oscillator Test

(Closed-book, no notes, real calculator no smartphones, 50 min)

Name: _______________________

Score:______/50

Question 1 Determine the frequency of oscillation in Hz for the circuit shown. The amplifiers have $R_i \to \infty$ and $R_o = 0$. Neglect loading effects of the two diodes, and you can assume the magnitude of the loop gain is sufficient to sustain oscillations. Show your work for full credit.

(15 points)

Solution For oscillation, the phase shift around the feedback loop must be $360^\circ$. The amplifier with gain $-10$ supplies $180^\circ$, so that each of the $RC$ sections must supply a $60^\circ$ phase shift. The frequency response and phase of a section are

$$T(j\omega) = \frac{1}{1 + j\omega RC}, \quad \phi = -\tan^{-1} \omega RC$$

Where $R = R_1 = R_2 = R_3$, and $C = C_1 = C_2 = C_3$. Thus

$$60^\circ = \tan^{-1} \omega RC \Rightarrow \omega RC = \sqrt{3} \Rightarrow \omega = \frac{\sqrt{3}}{RC} = 17.32 \times 10^3 \text{ rad/s}$$

Consequently $f_0 = 2.76$ kHz. Note: a SPICE simulation gives $f_0 = 2.98$ kHz. The difference between the calculated and SPICE values can be explained by the loading effect for the diodes. If we insert a buffer amplifier between the diodes and the last $RC$ stage, SPICE gives a frequency that matches our calculations.
Question 2  The analysis of a negative feedback oscillator shows that the loop gain transfer function is given by

\[ T(s) = \frac{A}{s^3 + as^2 + bs + 1} \]

where \(a\) and \(b\) are determined by the circuit components and \(A\) is the dc gain. Determine expressions in terms of \(a\), \(b\), and \(A\) for the oscillation frequency \(\omega_0\) (rad/s) and the minimum \(A\) required for sustained oscillation. (10 points)

Solution Set \(s = j\omega\) and \(T(j\omega) = -1\). Then

\[ \frac{A}{-j\omega^3 - a\omega^2 + jb\omega + 1} = -1 \]

\[ \frac{A}{(1 - a\omega^2) + j(b\omega - \omega^3)} = -1 \quad \cdots (1) \]

The RHS is real so the LHS should have no imaginary part, so that

\[ \omega^3 = b\omega \Rightarrow \omega = \sqrt[3]{b} \]

Substitute this in (1) to find

\[ \frac{A}{1 - ab} = -1 \Rightarrow A = ab - 1 \]
Question 3 For the oscillator shown, determine the amplitude and frequency (in Hz) of the output at $v_o$. Explain briefly the function and purpose of the SPICE statement

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.IC V(Vo) = 1u.
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You can assume that very little current flows through the diodes. Consequently, they will not have the normal 0.7 V across them when conducting. Rather, use $V_r = 0.35$ V. (15 points)

Solution

The frequency of oscillation is $f_0 = \frac{1}{2\pi RC}$ where $R = 4.7K$ and $C = 1$ nF. Substituting values give $f_0 = 33.9$ kHz. The purpose of the SPICE statement `.IC V(Vo) = 1u` is to set the voltage at node $V_o$ to 1$\mu$V. In oscillator circuits, where there is no input source, one has to provide an initial signal that is then fed back, amplified, etc. In this particular circuit, this statement is not needed, but it is good practice to always include this for oscillators.

Assume the amplitude has stabilized at $v_{om}$, and consider the instant when $v_o$ is at the crest of the sinewave. Since the current through the diodes is negligible (see problem statement), the voltage at the junction of $R_3$ and $R_2$ is $v_{om}/2$. The voltage at the inverting input of the amplifier is $v_{om}/2 - 0.35$. For a Wien bridge this voltage is one third of the output voltage, so that

$$\frac{v_{om}}{2} - 0.35 = \frac{v_{om}}{3}$$

Solving yields $v_{om} = 2.1$ V
**Question 4** Draw the small-signal model for the $LC$ oscillator shown. For the BJT, include $g_m$, $r_o$, $r_π$, $c_π$, and $c_μ$. You should also indicate $v_π$ along with its proper polarity. Label the collector, base, and emitter.

Next, briefly explain how you would go about determining the minimum $g_m$ required for oscillation. Do not find the condition, just explain a procedure. For example, you can start with something such as “Write a KVL equation around loop xxx and then solve for yyy. Next, set $s = j\omega$, and …”. You should not need more than 3–4 sentences. *(10 points)*