Question 1  (65 points)

The ac equivalent for a Colpitts oscillator is shown. The transistor is biased at \( I_{CQ} = 0.25 \text{ mA}, \beta = 100, R_E = 1\text{K}, L = 5\mu\text{H}, C_1 = 20\text{ pF}, C_2 = 100\text{ pF} C_3 \rightarrow \infty, \text{ and } C_4 = 0.01\mu\text{F}. \) Assume the transistor’s capacitances \( C_\mu, C_\pi \) can be ignored.

(a) Draw a small-signal model for the oscillator. (b) Show that the frequency of oscillation is

\[
\omega_0 = \frac{1}{\sqrt{L/C}} \sqrt{1 + \frac{L}{r_\pi R_E (C_1 + C_2)}}
\]

Let \( C = C_1 C_2 / (C_1 + C_2) \), which is the series combination of \( C_1 \) and \( C_2 \). The first part \( (1/\sqrt{LC}) \) is the familiar resonant frequency for an \( LC \) circuit, while the second part results from \( r_\pi \) in the circuit. By substituting the circuit values, show that

\[
\omega_0 = \frac{1}{\sqrt{LC}}
\]

is a good approximation. That is, the effect of \( r_\pi \) is small. (c) What is the minimum transconductance to sustain oscillation? (c) A variable capacitor \( C_3 \) is added to the circuit and has a range of 5–50 pF. What range of frequencies of oscillation can be achieved?

Notes. This is a challenging problem, but you can do it. Once you have the small-signal model, I strongly suggest you use the Method B (see lecture notes). Start by writing a KCL equation at the emitter.

Question 2. (a) The circuit shown is a negative impedance converter or NIC. Its input impedance/resistance is negative, makes it useful is a variety of circuits, as we will explore in the part of the problem. For now, show that the input resistance is \( -R_F \). (5 points) (b) Determine an expression for the frequency of oscillation for the oscillator below. What value for \( R_F \) is required for sinusoidal oscillation? (20 points)
Question 3. For the circuit below, \( C = 1 \) nF, and \( R = 4.7 \)K. Further, the power supply is \( \pm V_s = \pm 15 \) V, and \( R_4 = 20 \)K, \( R_3 = 3 \)K. The Si diodes have a 0.7 V turn-on voltage. Determine the frequency and amplitude of oscillation. (10 points)