**Homework Assignment 07**

**Question 1** For a buck regulator with smoothing capacitor with value $C$ and $ESR = 0$, the output ripple voltage is

$$
\Delta v_C = \frac{\Delta i_L}{8f_s C}
$$

(a) Derive an expression for the output ripple voltage amplitude for the case when $ESR \neq 0$ (8 points)

(b) Next calculate the ripple voltage for $f_s = 20 \times 10^3$ Hz, $\Delta i_L = 2$ A, $C = 22 \mu F$ and $ESR = 0.5 \Omega$ (4 points)

**Solution**

**Part (a)** The capacitor current approximately equals the time-varying component of the inductor current, namely $\Delta i_L$. This current develops a voltage $(\Delta i_L) \times ESR$. Note however, that this voltage is $90^\circ$ out of phase with the capacitor voltage. The total ripple voltage is then

$$
v_r = \Delta i_L \times ESR + j \frac{\Delta i_L}{8f_s C}
$$

$$
|v_r| = \Delta i_L \sqrt{ESR^2 + \left(\frac{1}{8f_s C}\right)^2}
$$

**Part (b)**

$$
|v_r| = \Delta i_L \sqrt{ESR^2 + \left(\frac{1}{8f_s C}\right)^2}
$$

$$
= \Delta i_L \sqrt{0.5^2 + \left(\frac{1}{8 \times 20 \times 10^3 \times 22 \times 10^{-6}}\right)^2}
$$

$$
= 1.15 V
$$
Question 2 (Buck Converter) (a) Design a buck switching converter to convert a 17 V input voltage to a 10 V output voltage. The converter must always be in continuous mode. The load current will be in the range 1–3 A. The inductor ripple current must be less than 2 A and output ripple voltage must be less than 100 mV. Specify the type of diode, switching frequency, duty cycle, and values for $L$ and $C$. For simplicity, assume a perfect switch, neglect diode voltage drop, and capacitor ESR. **(15 points)**

(b) Sketch the inductor current when the load current is 1 A. **(3 points)**

(c) Even though there is no feedback/regulation in the circuit, the output voltage will be approximately 10 V for a range of load resistances. What is this range? **(5 points)**

Solution

Use a Schottky diode and assume $V_F = 0.2$ V. For a buck converter with a perfect switch and a catch diode with on voltage $V_F$, we have

$$V_O = D - (-1D)V_F$$

$$17 = D - (1 - D)(0.2) \Rightarrow D = \frac{10.2}{17.2} = 0.593 \text{ (Duty Cycle)}$$

Alternatively, if one ignores the Schottky diode drop then $V_O \approx DV_l \Rightarrow D = 0.588$. Pick a switching frequency of $f_s = 20$ kHz, so that $t_{ON} = 0.593/(20 \times 10^3) = 29.7 \mu s$. The ripple current $\Delta i_L$ is 2 A. Assume $V_F \approx 0$. Use equation 11.43 in Franco:

$$L = \frac{V_O(1 - V_O/V_l)}{f_s \Delta i_L} = \frac{10(10 - 10/17)}{(20 \times 10^3)2} = 103 \mu H$$

For the ripple voltage

$$C = \frac{\Delta i_L}{8f_s \Delta V_C} = \frac{2}{8 \times 20 \times 10^3 \times 0.1} = 125 \mu F$$
Part (b)

As long as the converter is in continuous mode, the output voltage is independent of the load. Thus, for resistances less than \( \frac{10}{1} = 10 \, \Omega \) the output voltage is 10 V. For larger load resistances, the output voltage rises. In practice, losses in the inductor, diode, and switch limit the lowest load resistance.

Part (c)
**Question 3** Consider the gated-oscillator DC/DC regulator below. The hysteresis for $A_1$ is 20 mV, $R_1 = R_2 = 100K$, and the duty cycle $D$ of the oscillator is 30%.

a) What is the (mean) output voltage for an input voltage of 20 V? *(5 points)*  
b) What is the output ripple voltage? *(5 points)*

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**Solution**

**Part (a)** The feedback loop stabilizes when the mean voltage at the $R_1, R_2$ node matches the reference voltage. This is when

$$V_0 = \frac{R_1}{R_1 + R_2} \cdot \frac{V_0}{2} = 1.245 \text{ V}$$

$$V_0 = 2.49 \text{ V}$$

**Part (b)** As with the mean (dc) voltage, the ripple voltage at the output is twice the ripple at the comparator. Thus, the output ripple voltage is 40 mV.
**Question 4 (SC Converters)** The figure below shows the oscillator part of a SC DC/DC converter and the numbers in parenthesis are the IC pin numbers. The Schmitt trigger has thresholds 5.2 V and 2 V. Sketch the waveform at pin 7 if $I$ for the constant current generators is $0.5 \mu A$. Assume the BOOST control voltage keeps the corresponding switches open. On you figure indicate the maximum-, minimum-, average voltages of the waveform, as well as the period of the waveform. *(6 points)*

![Diagram](image)

**Solution**

As the capacitor charges/discharges with a constant current $I = 0.5 \mu A$, the voltage increases/decreases linearly at a rate

$$\frac{\Delta V_C}{\Delta T} = \pm \frac{I}{C} = \pm \frac{0.5 \mu A}{20 \text{ pF}} = \pm 25.6 \times 10^3 \text{ V/s}$$

The Schmitt trigger trip points are $5.2 - 2 = 3.2 \text{ V}$ apart and this is the amplitude of triangular voltage waveform at pin 7. It takes $\Delta T = 3.2/25.6 \times 10^3 = 125 \mu s$ for to charge/discharge the capacitor, so the period is $250 \mu s$ and the frequency is 4 kHz.
**Question 5 (SC Converters)** The figure below shows the LTC1144 SC IC configured to invert a voltage. Assume the switching frequency is 4 kHz, \(V^+ = 5\) V and the load at \(V_{OUT}\) (i.e., pin 5) is a 1K resistor. What are the minimum values for the capacitors to ensure \(V_{OUT}\) is within 10 mV of \(-5\) V? Assume the LTC1144’s internal switches has zero ON resistance. (6 points)

![Circuit Diagram]

**Solution**

The current through the load resistor is \(I_L = 5/1K = 5\) mA. During one cycle (250 \(\mu\)s) the resistor drains
\[
\Delta Q = (I_L)(\Delta T) = (5 \times 10^{-3})(125 \times 10^{-6}) = 625 \text{ nC},
\]
and the flying capacitor must replenish this. Now

\[
\Delta Q = C\Delta V \\
C = \frac{\Delta Q}{\Delta V} = \frac{625 \times 10^{-9}}{10 \times 10^{-3}} = 62.5 \mu F
\]
Question 6 (Switched Capacitors) In the switched capacitor circuit below, the voltages are $V_1 = 2$ V, and $V_2 = 1$ V, the capacitor value is $C = 10$ pF, and the clock frequency is $f_c = 100$ kHz.

(a) Determine the charge transferred from $V_1$ to $V_2$ during each clock pulse.
(b) What is the average current that source $V_1$ supplies?
(c) If the “on” resistance of each MOSFET is $1$ kΩ, determine the time required to transfer 99% of the charge during each half-clock period.

Solution

Part (a) \[ \Delta Q = C \Delta V = (10 \times 10^{-12})(2 - 1) = 10 \times 10^{-12} \text{ Coulomb} \]

Part (b) Current is defined as charge/unit time, and during each clock pulse ($10 \mu s$), a charge $10 \times 10^{-12}$ C is transferred from $V_1$. Thus, it supplies an average current:

\[ I_{ave} = \frac{\Delta Q}{\Delta t} = \frac{10 \times 10^{-12}}{10 \times 10^{-6}} = 1 \times 10^{-6} \text{ A} \]

Part (c)

Note that $Q = CV$, so that when 99% of the charge is transferred, the voltage is 99% of its final value. Thus:

\[ 1 - e^{-\frac{t}{RC}} = 0.99 \]
\[ t = 4.6 \times RC \]
\[ = 4.6 \times (1 \times 10^3)(10 \times 10^{-12}) = 46 \text{ ns} \]
**Question 7 (Principles)** The circuit has been in steady-state prior to the switch closure at $t = 0$. Determine and sketch the voltage $v(t)$ for $t > 0$.

**Solution** The circuit at $t = 0_-$ is shown in (a) below. The current through the inductor is easily found as $i_L(0_-) = 8/3 \, \text{A}$, and $v(0_-) = 16 \, \text{V}$. At $t = 0_+$, since the current through the inductor cannot change instantaneously, the circuit is equivalent to (b) below. Applying KCL gives $v(0_+) = 52/3 \, \text{V} = 17.33 \, \text{V}$. The circuit at $t = \infty$ is shown in (d) and it is clear the inductor shorts all but the $4 \, \Omega$ resistor and so that $v(\infty) = 24 \, \text{V}$. The circuit time constant is $L/\mathcal{E}_{\text{Th}}$, where $\mathcal{E}_{\text{Th}}$ is the Thevenin equivalent resistance the inductor sees, see (e) below. This is easily calculated as $\mathcal{E}_{\text{Th}} = 2 \, \Omega$ so that the time constant is 2 s.

Combining gives

$$v(t) = 24 - (24 - 17.33) e^{-\frac{t}{2}} = 24 - 6.66 e^{-\frac{t}{2}} \, \text{V}$$