55:141 Advanced Circuit Techniques

Switching Regulators

Material: Lecture Notes, Handouts, and Sections of Chapter 11 of Franco
Sidebar: Parts Per Million (PPM)

- Part Per Million an abbreviation for “parts-per-million” and is commonly used when very small changes from a nominal value of some sort is involved, for example temperature coefficients (TCRs) of precision resistors.
- To convert from ppm to %, divide the number by 100,000
- To convert from % to ppm, multiply by 100,000
- Thus, a TCR of 100 ppm/°C is the same as a TCR of 0.01%/°C
Resistors Specifications

- **Temperature Coefficient.** The resistance of a conductor changes with changes in temperature. This change is in general nonlinear, but for small temperature changes may be approximated as linear, and is commonly expressed as a *temperature coefficient* (TCR). The units are ppm/°C.

TRC is relevant for reversible changes in resistance as a function of temperature. Assume a TRC of +150 ppm/°C for a 1K resistor. If the ambient temperature rises from 25 °C to 30 °C, then the resistor’s value will change with

\[
\Delta R = 1,000 \times \frac{150}{1,000,000} (30 - 25) = 0.75 \, \Omega
\]

If the temperature drops to 25 °C the resistor’s value returns to 1K.
Why Study Power Supplies?

![Bar chart showing trends in billions for different years: CY2007, CY2008, CY2009, CY2010, CY2011, CY2012, CY2013. Categories include Amplifiers, Voltage regulators, Data converters, Interface ICs, Other analog. Source: Gartner (December 2009).]

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Why Study Power Supplies?

We will look at this...
Primary switching power supplies
- Convert line (120/240 VAC) to DC
- Switchers, SMPS, medical switchers, etc.
- Efficient
- Small
- Uses smaller transformer, → inexpensive compared to a non-switching power supply with the same power-handling capability

Cell-phone chargers, laptop power supplies, medical equipment, etc.

Secondary switching power supplies
- Convert DC to DC
- Also called DC/DC converters
Linear Power Supply

Bulky, expensive transformer

Ripple voltage is

\[ V_{pp} \leq \frac{I_L}{2fC} \]

Where \( f \) is the line frequency (50/60 Hz)

Thus for good pre regulation, need large \( C \) (bulky, $$$)

Linear regulator, often dissipates significant power and need expensive heat sinks.
Switch Mode Power Supply

The rest of the circuit can tolerate significant ripple, so $C$ can be small.

Recall the universal transformer equation:

$$E_{\text{rms}} = \frac{2\pi fNaB_{\text{peak}}}{\sqrt{2}} \approx 4.44fNaB$$

Ripple voltage is

$$V_{pp} \leq \frac{I_L}{2fC}$$

Chop DC at 50 kHz – 1 MHz

High frequencies require small $C$

High frequencies require smaller transformers
Some Basic Circuit Theory

Differential equation for a capacitor

\[ i_C(t) = C \frac{dv_C(t)}{dt} \]

The voltage across a capacitor cannot change instantaneously. “Instantaneously” means \((dv_C/dt) \to \infty\), so would mean \(i_C \to \infty\), which is physically impossible.

Consider a capacitor that is initially charged to \(V_0\) V, that is then connected to a battery with voltage \(V_m\) through a resistor \(R\). When the switch is closed, the voltage right after the switch is closed is still \(V_0\). The capacitor charges exponentially to \(V_m\) with a time constant \(\tau = RC\).

\[ v_c(t) = K_1 + K_2 e^{-t/\tau} \]
Some Basic Circuit Theory

Differential equation for an inductor

\[ v_L(t) = L \frac{di_L(t)}{dt} \]

The faster the current through an inductor changes, the bigger the voltage across the inductor: if \((di_L/dt) \to 0\), then \(V_L \to \infty\). This idea is used to generate high voltages, for example ignition systems in our motor vehicle.

In transient circuits, we say that the current through an inductor cannot change instantaneously.

In steady state, with switch open, current through inductor is 1 A

Right after switch is closed, the inductor behaves as a 1 A current source.
Some Basic Circuit Theory

**Capacitors**

\[ i(t) = C \frac{dv(t)}{dt} \]

- dc → \( dv(t)/dt = 0 \) → \( i = 0 \)

Capacitor is an “open circuit to dc”

dc circuits, replace capacitors with open circuits

**Inductors**

\[ v(t) = L \frac{di(t)}{dt} \]

- dc → \( di(t)/dt = 0 \), \( v = 0 \)

Inductor is a short to dc

Dc circuits, replace inductors with short circuits

\[ \frac{dv}{dt} = 0 \]

Capacitor is an “open circuit to dc”

Dc circuits, replace capacitors with open circuits
Back to Basics

\[ v_L = L \frac{di}{dt} = L \frac{\Delta I}{\Delta t} \]
Relay inductance: 0.3 H, 150 Ω resistance

About 60 mA flows when the transistor is on

What happens when the transistor turns off?

Answer: the collapsing magnetic field induces a voltage with the polarity shown. The magnitude is

$$v_L = L \frac{di}{dt}$$

Assume the transistor turns off in 0.1 ms. Then

$$v_L = 0.3 \frac{60 \times 10^{-3}}{0.1 \times 10^{-3}} = 180 \text{ V}$$

Is 0.1 ms for turn-off a good assumption?
This diode provides a low impedance path and protects the transistor from the voltage spike.

This diode provides a low impedance path and protects the rest of the circuit from the voltage spike.

Diode will not work, since it will conduct on alternate cycles. Use RC “snubber”
DC/DC Converter

Linear Regulator

Switching Regulator
Switching Regulator Topologies

- **Buck**
  - \( V_O = D V_I \)

- **Boost**
  - \( V_O = \frac{1}{1 - D} V_I \)

- **Buck-Boost**
  - \( V_O = -\frac{D}{1 - D} V_I \)

Note that the average output voltage is essentially independent from the value of \( L \) and \( C \).
Assume the switch is driven by a square wave with period $T$ and cycle $D$. The ON time is then $t_{ON} = DT$ and the OFF time is $t_{OFF} = (1 - D)T$.

We will assume $V_O$ is constant and $V_O > V_I$.

During $t_{ON}$, voltage across $L$ is $V_I$, and

$$\Delta i_L(ON) = t_{ON} \frac{\Delta V_L}{L} = DT \frac{V_I}{L} \quad \text{(increase)}$$

During $t_{OFF}$, the voltage across $L$ is $V_O - V_I$, and

$$\Delta i_L(OFF) = t_{OFF} \frac{V_I - V_O}{L} = (1 - D)T \frac{V_I - V_O}{L} \quad \text{(decrease)}$$

In steady state $\Delta i_L(ON) = \Delta i_L(OFF)$. Combining equations leads to

$$V_O = \frac{V_I}{(1 - D)}$$
Buck Converter

What factors affect efficiency?

\[ D = \frac{t_{ON}}{t_{ON} + t_{OFF}} = \frac{t_{ON}}{T_S} = f_s t_{ON} \]
Assume output and input voltages are constant. $V_{\text{SAT}}$ is the voltage across the (BJT/FET) switch when closed, and $V_D$ is the voltage across the diode when conducting.

With switched closed, voltage across the inductor is $V_I - V_o - V_{\text{SAT}}$ and the current through the inductor increases linearly. After $t_{\text{ON}}$, the inductor current has increased by

$$\Delta i_L = \frac{V_o - V_I - V_{\text{SAT}}}{L} t_{\text{ON}}$$

When the switch opens, current cannot change instantaneously, voltage across inductor reverses, and cathode swings to below ground. The voltage across the inductor is $V_o + V_D$ and the current decreases linearly as the magnetic field collapses. After $t_{\text{OFF}}$, the current has decreased by

$$\Delta i_L = \frac{V_o + V_D}{L} t_{\text{OFF}}$$

In the steady state, the increase during $t_{\text{ON}}$ should balance the decrease during $t_{\text{OFF}}$, so that

$$\frac{V_o - V_I - V_{\text{SAT}}}{L} t_{\text{ON}} = \frac{V_o + V_D}{L} t_{\text{OFF}}$$

Solving for $V_o$ and recognizing that $t_{\text{ON}} + t_{\text{OFF}} = 1/f$ and that $D = t_{\text{ON}} + t_{\text{OFF}}$ is the duty cycle $D$, then

$$V_o = D(V_I - V_{\text{SAT}}) - (1 - D)V_D \approx DV_I$$

Note that this is independent of the load current, $L$, and $C$.
Called *continuous conduction mode* (CCM), where the inductor current never goes to zero.

In the CCM, output voltage is independent of the load current.
Buck Regulator: Discontinuous Mode

\[ \Delta i_L = \frac{V_o - V_I - V_{sat}}{L} t_{ON} \]

\[ \Delta i_L = \frac{V_o + V_D}{L} t_{OFF} \]

When load current drops, the output voltage stays constant, and \( \Delta i_L \) stays the same.

When the load current falls below a critical value \( I_{omin} \) the inductor goes to zero during a cycle.

This is called **discontinuous conduction mode (DCCM)**

\[ I_{omin} = \frac{T}{2L} V_o \left(1 - \frac{V_o}{V_i}\right) \]

In DCCM, more energy is stored than extracted in each cycle and the output voltage rises (and some of our assumptions thus far are not valid…)

The controller will reduce \( D \) to keep \( V_o \) fixed
Buck Switching Regulator

When the converter goes into DCCM, output voltage rises and controller reduces duty cycle to keep voltages fixed.

\[ V_O = D V_I \]

Only valid for CCM
Capacitor smoothes the output voltage. The capacitor charge current is $I_L - I_o$.
The charge applied and removed during one cycle corresponds to the hatched area (remember $Q = I \times t$). The change in capacitor voltage is thus

$$\Delta V_o = \frac{\Delta Q}{C} = \frac{1}{2} \left( \frac{t_{ON}}{2} + \frac{t_{OFF}}{2} \right) \frac{\Delta I_L}{2} = \frac{T}{8C} \Delta I_L$$

Note: both $\Delta I_L$ and $\Delta V_o$ are peak-to-peak values.
Buck Ripple Voltage

However, the capacitor has an ESR, that also contributes to the ripple voltage. The voltage across the ESR is

\[ \Delta V_{ESR} = ESR \times \Delta i_C \]

Note that the voltage is \textit{not} in phase with the voltage across the capacitor:

\[ V_r = \Delta v_{ESR} + j \Delta V_o \]
Buck Capacitor Selection

\[ \Delta V_o = \frac{\Delta Q}{C} = \frac{1}{C} \left( \frac{t_{ON}}{2} + \frac{t_{OFF}}{2} \right) \frac{\Delta I_L}{2} = \frac{T}{8C} \Delta I_L \]

\[ \Delta V_{ESR} = ESR \times \Delta i_C \]

\[ V_r = \Delta V_{ESR} + j\Delta V_o \]

Both capacitance and ESR contribute to ripple voltage.

Capacitor dissipates \( ESR \times i^2_{C(rms)} \) and must be able to handle this.

Capacitor must be able to handle ripple current \( \Delta I_L \)

Good capacitor can cost $$$
Buck Coil Selection

Continuous Mode

Assume $V_{\text{SAT}} = V_D = 0$

$\Delta i_L = \frac{V_o - V_I}{L} t_{ON}$

$t_{ON} = \frac{L \Delta i_L}{V_o - V_I}$

$t_{OFF} = \frac{L \Delta i_L}{V_o}$

$\Delta i_L = 0.2I_L$

Suggested Design Equation

Coil must handle $I_P$ without saturating core

$\Delta i_L = \frac{V_o - V_I - V_{\text{satt}}}{L} t_{ON}$

$\Delta i_L = \frac{V_o + V_D}{L} t_{OFF}$

$t_{ON} + t_{OFF} = 1/f_s$

$L = \frac{V_o (1 - V_o / V_I)}{f_s \Delta i_L}$
Efficiency

- If all the components are lossless:
  - ESR = 0
  - VD = 0
  - ESL = 0
  - Switch resistance = 0
- Then the buck DC/DC converter can provide 100% conversion

\[
\eta = \frac{P_o}{P_o + P_{\text{diss}}}
\]

\[
P_o = V_o I_o
\]

\[
P_{\text{diss}} = P_{SW} + P_D + P_{\text{coil}} + P_{\text{cap}} + P_{\text{controller}}
\]
Efficiency

\[ \eta = \frac{P_o}{P_o + P_{diss}} \]

\[ P_o = V_o I_o \]

\[ P_D = V_D I_{F(avg)} + f_s (V_R I_F t_{RR}) \]

\[ P_{diss} = P_{SW} + P_D + P_{coil} + P_{cap} + P_{controller} \]

\[ P_D = V_{SAT} I_{SW} + 2 \Delta v_{SW} \Delta i_{SW} \Delta t_{SW} \]

Conduction loss  Switching loss

\[ P_{cap} = ESR \times I_{C(rms)}^2 \]

\[ P_{coil} = R_{coil} \times I_{L(rms)}^2 + P_{core}(f) \]
Buck Coil Selection

\[ w_L = \frac{1}{2} L i_L^2 \]

\[ L = \frac{V_o (1 - V_o / V_I)}{f_S \Delta i_L} \]

Suggested Design Equation

\[ \Delta i_L = 0.2 I_L \]
Coil Selection

Coil must carry some rms current $I_L$ to feed the load.

Coil must handle $I_P$ without saturating the core.

- **Buck**: $I_L = I_O$
- **Boost**: $I_L = \frac{V_O}{V_I} I_O$
- **Buck-boost**: $I_L = \left(1 - \frac{V_O}{V_I}\right) I_O$

**Continuous Mode**

**Discontinuous Mode**
Both Pulse Frequency Modulation and Pulse Width Modulation (PWM) can be used for control.

Most commercial ICs use PWM control with $f_s$ between 10 kHz and 1 MHz.

Two-types of control: voltage and current-mode control.
Voltage Mode PWM Control

Sawtooth generator running at $f_s$

Question: what type of feedback? How does it affect the output resistance?
Gated Oscillator Converter

- Reference
- Comparator
- Oscillator, fixed duty cycle
- Uncommitted op-amp
- Switch

Diagram showing the components of a gated oscillator converter:
- \( V_{IN} \) to 1.245V Reference
- Reference to Comparator (A1)
- Comparator (A1) to Oscillator
- Oscillator to Uncommitted op-amp
- Uncommitted op-amp to Switch
- Switch to \( V_{IN} \)

Components:
- 1.245V Reference
- Comparator
- Oscillator
- Uncommitted Op-Amp
- Switch
Gated Oscillator Converter

Comparator with hysteresis ~ 10 mV
Non-linear element → non-linear feedback
More stable feedback, more efficient converter
Very low supply current – oscillator switched only when needed, when FB drops below reference

Adaptive base drive to make sure switch is not overdriven \(\rightarrow\) improve efficiency

Hysteresis ensures loop stability without complex compensation networks

\(D\) Duty cycle \(\rightarrow\) optimized for circuits where \(V_{in}\) and \(V_{out}\) differ by factor \(D\)
LT1173 DC/DC Converter

- Uncommitted op-amp
- 1 A internal switch
- Switch frequency ~ 24 kHz, duty cycle ~ 50%
- Comparator with ~ 5 mV hysteresis
- Comparator
- 50% Duty cycle
- 100 µA in standby mode
- Step-up or step down
- Supply 2.0 – 30 V
Wiring as a buck regulator
Choose either $R_2$ or $R_1$, solve for other one. Choose a value $> 50K$. 

$$V_o \cdot \frac{R_1}{R_1 + R_2} = 1.245$$
Choose either $R_2$ or $R_1$, solve for the other one. Choose a value > 50K.
\[ \frac{V_o}{R_1 + R_2} = 1.245 \]

Choose either \( R_2 \) or \( R_1 \), solve for the other one. Choose a value > 50K.
Choose either $R_2$ or $R_1$, solve for the other one. Choose a value $> 50K$. 

\[
\frac{V_o}{R_1 + R_2} = 1.245
\]
Choose either $R_2$ or $R_1$, solve for other one. Choose a value > 50K.

$$V_o \frac{R_1}{R_1 + R_2} = 1.245$$

$C = \text{Low ESR}$

$D = \text{Schottky}$

$L = \text{No saturation}$
Choose either $R_2$ or $R_1$, solve for other one. Choose a value $> 50K$. 

$$V_o \frac{R_1}{R_1 + R_2} = 1.245$$

Wiring as a boost regulator:

$$V_o = \frac{1}{1 - D} V_I$$

$C = \text{Low ESR}$

$D = \text{Schottky}$

$L = \text{No saturation}$

$$\text{Choose either } R_2 \text{ or } R_1, \text{ solve for other one. Choose a value } > 50K$$
Choose either $R_2$ or $R_1$, solve for the other one. Choose a value $> 50K$.

$$V_o \frac{R_1}{R_1 + R_2} = 1.245$$
Increasing Output Drive

What if this switch can not handle the peak and rms currents?

Buck Regulator
One can use the internal switch to control a more suitable external BJT or FET.
LT1173 DC/DC Converter

**Figure 5. Step-Up Mode Hookup.**
Refer to Table 1 for Component Values

**Figure 6. Step-Down Mode Hookup**

**Figure 7. Positive-to-Negative Converter**
Flyback Regulator

Triple-output flyback regulator

Diagram of a flyback regulator with components labeled as follows:
- $V_{SW}$
- $V_C$
- GND
- $FB$
- $R_1$ = 1.24 kΩ
- $R_2$ = 3.74 kΩ
- $L_0$
- $L_1$
- $L_2$
- $L_3$
- $D_1$
- $D_2$
- $D_3$
- $D_4$
- $D_5$
- $C_1$
- $C_2$
- $C_3$
- $C_4$
- $C_5$
- $V_I$
- $+12$ V
- $-12$ V
- $+5$ V
- $3$ V to $20$ V
Switched Capacitor Converters

- Also called *charge-pump* converters
- *Flying capacitor* converters
- Generally used for low-power (~ 100 mA or less)
Switched Capacitor Converters

Basic Circuit

“Flying Capacitor”

Non-overlapping switches

Switch frequency 10–100 kHz

V\text{IN} \quad S1 \quad S2

C1

\quad S3 \quad S4

V\text{out} = - V\text{IN}

C2

Switch frequency 10–100 kHz
Switched Capacitor Converters

Voltage Inverter

Step 1: Charge C1 to $V_{in}$

Step 2: All switches open

Step 3: Transfer charge

With proper capacitors and switches, the inversion efficiency can be close to 100%
Carefully note the polarity of this capacitor. It seems wrong with the + side connected to ground, but remember, pin 5 is at -6V.

(1) Assume 6 V Here

(2) Then -6 V here
FIGURE 23. COMBINED NEGATIVE CONVERTER AND POSITIVE DOUBLER

\[ V_{\text{OUT}} = -(nV_{\text{IN}} - V_{\text{FDX}}) \]

\[ V_{\text{OUT}} = (2V^+) - (V_{\text{FD1}}) - (V_{\text{FD2}}) \]
More Circuits

FIGURE 19. CASCADING DEVICES FOR INCREASED OUTPUT VOLTAGE

(1) Assume 6 V Here

(2) Then -6 V here

(3) Which is the $V_{dd}$ for this one

(4) The GND is the $V+$ supply for this one

(5) So here we get -12 V
More Circuits

- Increase output current capacity
- Noisy output
- NOR gate helps synchronize switching and reduce output ripple noise
Switched Capacitor Building Block

Charge transferred each cycle

\[ \Delta q = C_1 \Delta V \]

Charge transferred per unit time is

\[ \frac{\Delta q}{T} = f_s C_1 (V_1 - V_2) = I \]

Rewriting

\[ I = \frac{(V_1 - V_2)}{1/(f_s C_1)} \]

\[ I = \frac{(V_1 - V_2)}{R_{eq}} \]

\[ R_{eq} = \frac{1}{f_s C_1} \]

\[ I_c = C \frac{dV_c}{dt} \]

\[ Q = CV \]

\[ \Delta Q = C \Delta V \]
The switches are commonly implemented with FETs. While SC ideas have been around for a long time, modern IC manufacturing allows for commercially-viable implementations.

The switches are not ideal and has an “on” resistance of a few Ohms.
\[ R_1 = \frac{1}{f_s C_1} \]

Very important – in ICs we can control the ratio of \( C_1 / C_{F1} \) very well.

We can adjust the filter bandwidth with a clock.

This allows for controlling filter with a microcontroller.
Switched Capacitor Building Block

\[ R_1 = \frac{1}{f_s C_1} \]

\[ R_2 = \frac{1}{f_s C_2} \]

\[ B_1 = \frac{f_s}{2\pi} \frac{C_1}{C_{F1}} \]

\[ B_2 = \frac{f_s}{2\pi} \frac{C_2}{C_{F2}} \]
Assume $C_A$ and $C_B$ are initially uncharged. When the switch is thrown to (a), $C_A$ charges and attains a charge $C_A V_i$. When the switch is thrown to (b) the capacitors are in parallel with value $C_T = C_A + C_B$, and charge $C_A V_i$ on them, so the output voltage after the first cycle is

$$V_1 = V_i \frac{C_A}{C_T}$$

The charge on $C_B$ after the first cycle is

$$V_1 C_B = V_i \frac{C_A}{C_T} C_B$$

When the switch is returned to (a), $C_A$ again attains a charge $C_A V_i$. When the switch is thrown to (b), the capacitors are in parallel and has a charge and output voltage after the second cycle:

$$Q_2 = V_i C_A + V_i \frac{C_A}{C_T} C_B$$

$$V_2 = V_i \frac{C_A}{C_T} + V_i \frac{C_A C_B}{C_T C_T}$$

One can see that the output voltage is a geometric series

$$V_n = V_i \frac{C_A}{C_T} \left( 1 + \frac{C_B}{C_T} + \frac{C_B^2}{C_T^2} + \cdots + \frac{C_B^{n-1}}{C_T^{n-1}} \right)$$

or

$$V_n = V_i \frac{C_A}{C_T} \left( \frac{1 - (C_B / C_T)^n}{1 - C_B / C_T} \right)$$
Let $C_A = C_B$. How many cycles are required for the output voltage to be within 99% of $V_i$?

**Answer**

$$V_n = V_i \frac{C_A}{C_T} \left( \frac{1 - (C_B/C_T)^n}{1 - C_B/C_T} \right)$$

$C_A = C_B$, so that $C_A/C_T = C_B/C_T = 1/2$ and with the 99% requirement, this equation becomes

$$0.99 = \frac{1}{2} \left( \frac{1 - (1/2)^n}{1 - 1/2} \right) = (1 - 0.5^n)$$

Solving for $n$ yields $n = 6.64$ so that after 7 cycles the output voltage is within 99% of the input voltage.
Switched Capacitor Subtractor

\[ C_S = C_H \]

**Step 1**
- \( C_S \) charges to \( V_2 - V_1 \)
- Charge on \( C_S \) is \( Q_D = C_S(V_2 - V_1) \)

**Step 2**
- \( C_T = C_S || C_H = C_S + C_H \)
- \( V_H = Q_D / C_T = C_S(V_2 - V_1)/(C_S + C_H) \)
- \( = 0.5(V_2 - V_1) \)

**Step 3**
- \( C_S \) charges to \( V_2 - V_1 \)
- Charge on \( C_S \) is \( Q_D = C_S(V_2 - V_1) \)

After a few cycles, \( V_H = (V_2 - V_1) \)
- Output voltage is \( V_O = (V_2 - V_1)(R_2/R_1 + 1) \)
The accuracy of the subtraction depends on the stray capacitances of the switches. IC manufacturing technology allows one to keep this quite small, and one can further minimize stray capacitance effects by using large value capacitors, 10 nF — 10 μF.
Switched Capacitor Instrumentation Amplifier

**Instrumentation Amplifier**

- **LTC1043 IC**
- Good CMRR

**CMRR vs Frequency**

- CMRR > 120dB at DC
- CMRR > 120dB at 60Hz
- Dual Supply or Single 5V
- Gain = 1 + R2/R1
- Vos = 150mV
- $\Delta V_{os} = 2 \mu V^\circ C$
- Common mode input voltage includes the supplies
When the switches are in the position shown, capacitor $C_1$ charges up to $V_1 - V_2$ and accumulates a charge $C_1(V_1 - V_2)$. 
The op-amp and $C_2$ is an integrator with

$$v_o = -\frac{1}{C_2} \int_0^t t \, dt = -\frac{C_1}{C_2} f_s \int_0^t (V_1 - V_2) \, dt$$

This is a difference integrator.

When the switches are thrown, both ends of $C_1$ is at ground and the charge is transferred to $C_2$. One can consider the switched capacitor as a current source with value

$$I = \frac{C_1 (V_1 - V_2)}{T} = C_1 (V_1 - V_2) f_s$$

where $f_s$ and $T$ are the switching frequency and switching period respectively.