Homework Assignment 03

Question 1 (75 points)

Overview

**Tuned Radio Frequency** (TRF) receivers are some of the simplest type of radio receivers. They consist of a parallel \( RLC \) bandpass filter with bandwidth \( BB \). The resonance frequency is \( 1/\sqrt{LC} \), and is adjustable. Typically \( C \) is made adjustable, but one could also adjust \( L \). By adjusting or tuning \( C \), one can place the resonance frequency right at the center of a radio station’s transmit frequency, and pick out the desired radio signal from all the other stations, as well as background noise. For example, in the Iowa City area, one can receive several radio stations in the Medium Wave (MW) band. In the U.S., MW covers the range 535 kHz to 1,705 kHz. The stations are separated by 10 kHz. The bandwidth is on the order of \( \pm 5 \) kHz. One of the stations is the WSUI radio station, owned by the University of Iowa, and it transmits an AM signal at 910 kHz.

A TRF receiver will pick out the WSUI signal by tuning the \( RLC \) circuit’s resonance frequency to 910 kHz. By adjusting \( C \), one can “slide” the \( RLC \) bandpass filter over the MW band and pick out other stations. The bandwidth of the \( RLC \) bandpass filter is determined by the bandwidth of the transmitted signal. Ideally \( B \) should be exactly the same as that of the incoming signal. If \( B \) is larger, then additional noise enters the receiver, while if \( B \) is too small, some of the signal information is lost. With AM transmission, the bandwidth is often quite small. With speech this is not a major problem, but music sound quality suffers. It is not possible to design a bandpass filter with a “brick wall” response, but with proper attention to details one can keep the \( RLC \) bandpass filter narrow enough so that it has the required selectivity to pick out the various stations.

Older radios used mechanical capacitors for tuning; modern radios almost exclusively use varactor diodes. Any \( pn \)-junction diode exhibits voltage dependent capacitance when reverse-biased. One can place a reverse-biased capacitor in parallel with an inductor and form a voltage-variable bandpass filter. Varactor diodes are diodes optimized with respect to voltage-dependence of their reverse-biased capacitance. For example, it is desirable that a small change in voltage results in a large change in capacitance. Also, the capacitance should make sense for the frequency range of interest. If the values are very small, this may require large value inductors \( \left( f_0 = 1/\sqrt{LC} \right) \). Further, the reverse leakage current should be small, since this can negatively impact the \( Q \) of the \( RLC \) circuit.

Once selected by the TRF front end, the signal is further processed. A typical scenario is an RF amplification stage(s), followed by a detector or demodulation stage, and then some audio amplification.

**Design Requirements**

For this exercise, students are to design a TRF front end consisting of a \( RLC \) bandpass filter that is tunable via a varactor. The requirements for the front end are

- Assume a +9V power supply is available
Use one of a selection of forms available for the inductor
  - Plastic pill bottles (stop by the lab to see what I have)
  - PVC pipe fittings (stop by the lab to see what I have)

Students should hand in a complete design that considers the following

- RFC specifications
- Required $Q$ for the inductor
- Output impedance. We need to know this to make sure the input impedance of the amplifier (which we will design later) does not ruin the selectivity.
- AWG gauge. Stop by the lab to see the available wire sizes.
- Type of form for the coil, number of turns, etc.
- Students should also describe how they will measure the $Q$ of their inductor at a frequency of 1.6 MHz.
- Use the NTE618 varactor

Remember, this is not just a paper exercise. The objective is to eventually build the circuit. You should carefully document your design decisions. There is no need to be verbose, but you should include all details.

Solution

The figure below shows the design.

Since this is a design, there is not a single solution. One possible approach is as follows. First, recall that $f_0 \propto 1/\sqrt{LC}$. The 535 kHz to 1.705 kHz range is approximately 1:3, so that the NTE 618 varactor capacitance should change over a 1:9 ratio. The graph below, constructed with data from the datasheet, shows that when varying with the varactor voltage between 1.2 and 6 V, the capacitance will change between 420 pF and 45 pF, which is approximately a 1:9 ratio.
We obtain the inductor value when the varactor voltage is 1.2 V. This corresponds to 535 kHz and 420 pF. We can of course use other voltages and corresponding capacitances. Thus

\[(2\pi)(535 \times 10^3) = \frac{1}{\sqrt{L(420 \times 10^{-12})}} \Rightarrow L = 211 \mu H\]

The next step is to design the inductor: diameter of form, wire diameter, number of turns, etc. A rule of thumb is that inductors with length to diameter ratio approximately 1 tend to have the best \(Q\). Also, one would want to use thicker wire (to counter skin effect) but because of the larger diameter, thicker coils are longer. Thinner wire is easier to work with, and will give more compact coils, but have higher resistance. Also, if the wire is too thick, it becomes difficult to wind the coils by hand. Taking these considerations into account, we pick an AWG 22 wire with 0.0253” diameter. Using an online tool for air core coils, we find that 78 turns will result in a coil 1.97” long with an inductance of 212 \(\mu\)H.

It is difficult to achieve \(Q_s\) of more than 100 if we don’t use Litz wire, so we will assume the \(Q\) of the coil is 100. This means the parallel resistance at 1.6 MHz is

\[R_p = Q\omega L = (100)(2\pi)(1.6 \times 10^6)(212 \times 10^{-6}) \approx 200K\]

This is \(R_{out}\) in the schematic if we ignore the losses in the capacitor. We can use a 10K potentiometer to provide a voltage that varies over a 1.2–6 V range. To make sure that this does not load the circuit and spoil the resonant tank’s \(Q\), we can insert an RFC between the varactor and the potentiometer wiper. The reactance of the RC should be much higher than \(R_p\). One can also use a large value (several MΩ) resistor, \(R_{bias}\) in the circuit. This work because the varactor is a reverse-biased \(pn\)-junction with little current flowing into it.

The input resistance of the amplifier should be also be much larger than \(R_p\) and the amplifier’s \(C_{in}\) should be smaller than the smallest value the varactor will produce (45 pF).
Question 2

Consider a 100 MHz source with 50 Ω internal resistance. Design a tapped capacitor resonant transformer for maximum power transfer to a 2K load. The network $Q_T$ should be 20 and assume that the inductor $Q$ is 100. Verify your design with a SPICE simulation. (25 points)

Solution

The inductor $Q$ is high so we will initially ignore it. For maximum power transfer, the capacitive transformer will transform the 50 Ω to 2K so that at the load, the equivalent resistance is 1K. The desired network $Q$ is 20, which corresponds to a bandwidth of $B = 100 \text{ MHz} / 20 = 5 \text{ MHz}$. This determines the resonator capacitance and inductance

$$B = \frac{1}{2\pi RC} \Rightarrow 5 \times 10^6 = \frac{1}{2\pi (1 \times 10^3)C} \Rightarrow C = 31.8 \text{ pF}$$

$$L = \frac{1}{(2\pi f_0)^2 C} = 79.58 \text{ nH}$$

The transformer ratio is

$$n = \sqrt{\frac{50}{1 \times 10^3}} = 0.158 = \frac{C_1}{C_1 + C_2}$$

Further

$$C = \frac{C_1C_2}{C_1 + C_2} = 31.8 \text{ pF}$$

Solving for $C_1, C_2$ yields $C_1 = 37.81 \text{ pF}$ and $C_2 = 201.3 \text{ pF}$.

To see how well the capacitive transformer will work, we check $nQ_T Q_E$. If this metric is larger than 20 the transformer is good, and if it is larger than 100, it is very good (see lecture notes).

$$Q_E = \omega_0 R_s (C_1 + C_2) = (2\pi)(100 \times 10^6)(50)(37.81 + 201.3) \times 10^{-12} = 7.5$$

$$Q_T = \frac{Q}{\omega_0 R_s C / n^2} = (2\pi)(100 \times 10^6)(50)(31.8 \times 10^{-12})/0.1586^2 = 40$$

$$nQ_T Q_E = 47.5$$

The $nQ_T Q_E = 47.5$ metric shows that the basic transformer is good.
However, thus far we neglected the series resistance of the inductor. An inductor $Q, Q_L = 100$, suggests this would be a valid approximation, but is it? A $Q_L = 100$, means that it has a series resistance $R_s = \omega_0 L / Q_L = 0.5 \, \Omega$. At resonance, the inductor and its series resistance will appear as a parallel inductor and resistor pair. The parallel inductor value is essentially the same as the series value, which can be verified by applying $L_P = L_s (1 + Q_L^2) / Q_L^2$. The series resistance appears as a parallel resistor with value

$$R_P = R_s (1 + Q_L^2) = 0.5 (1 + 100^2) \approx 5K$$

We designed the transformer to transform $50 \, \Omega$ to a $2K$ resistor, but the inductor places a $5K$ resistor in parallel with the load. How should we deal with this? One method is to redesign the transformer, but now with a load $R'_L = 5K || 1K$. This will give a different value for the inductor, which will then result in a different resistance in parallel with the $1K$ load. A Matlab script can be used to quickly iterate to a solution.

Another approach would be to simulate the transformer and see what impact the inductor losses have. Below is the output from such a simulation. It shows the voltage across the load as well as the power dissipated across the load. The center frequency of the transformer is $100 \, MHz$ as per the design. The bandwidth was obtained by using the cursors in Micro-Cap SPICE and is $5.5 \, MHz$. This is close to the $5 \, MHz$ and the $10\%$ difference is attributable to the losses in the inductor. In many cases, the transformer would be sufficient. If not, then one can tweak the design using the iterative approach outlined above.