Question 1  Part Per Million (ppm) is an abbreviation for “parts-per-million” and is commonly used when very small changes from a nominal value of some sort is involved.

(a) An 307.2-kHz crystal oscillator has a rated frequency precision of $\pm 20$ ppm. What is the corresponding precision in %? What is the corresponding variation in Hz? (b) An engineer is charged with finding a crystal for use in a 32.768 kHz oscillator. The crystal frequency stability should be $\pm 0.06$ Hz over $-40 \degree C$ to $85 \degree C$. How difficult will it be to meet this specification? Be sure to motivate your answer properly.

Solution

Part (a)

\[
\text{Precision} = \frac{20}{10^6} = 20 \times 10^{-6} \% = 0.0002\%
\]

\[
\text{Variation} = \pm \left( \frac{20}{10^6} \right) \left( 0.3072 \times 10^6 \right) = \pm 6.144 \text{ Hz}
\]

Note how we express ppm as $20/10^6$ and 307.2-kHz as $0.3972 \times 10^6$. This allows cancelation of the $10^6$ factor and eases calculations.

Part (b)

A specification of 32.768 kHz $\pm 0.06$ Hz translates to

\[
\text{Stability} = \pm \frac{0.06}{0.032768 \times 10^6} \times 10^6 \text{ ppm} = \pm 1.83 \text{ ppm}
\]

A search of 32.768 kHz crystal at electronic component vendors suggests that crystals with 5 ppm are the best COTS parts available. A more extensive search may locate a suitable crystal or it may be possible to have a custom crystal manufactured. Regardless, it will be difficult to meet the specification.
Question 2 Shown in an RF Electronics circuit. By circling and labeling, identify the following components or subcircuits: (a) Capacitive resonant transformer, (b) varactor, (c) npn BJT, (d) pnp BJT, (e) Zener diode, (f) trimmer capacitor. If there are multiple instances of a component/subcircuit, identify only one. (1 point each)
**Question 3** Two identical amplifiers are connected in cascade as shown. For the amplifiers, \( h_{11} = 1\text{K}, \ h_{12} = 1.5 \times 10^{-3}, \ h_{21} = 100, \) and \( h_{22} = 100 \mu S. \) Using two-port network theory, determine the overall voltage gain \( V_2/V_g. \) (20 points)

*Note:* you are strongly encouraged to use computer software such as Matlab to work this problem.

**Solution**

Consulting a table for two-port parameter conversion reveals that the \( ABCD \) parameters for an amplifier is

\[
\begin{bmatrix} A & B \\ C & D \end{bmatrix} = -\frac{1}{h_{21}} \begin{bmatrix} \Delta h & h_{11} \\ h_{22} & 1 \end{bmatrix} = \begin{bmatrix} 500 \times 10^{-6} & -10 \\ -1 \times 10^{-6} & -10 \times 10^{-3} \end{bmatrix}
\]

where \( \Delta h \) is the determinant of the \( h \)-parameter matrix. The \( ABCD \) matrix for the cascaded amplifier is the product of the individual \( ABCD \) matrices:

\[
\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 500 \times 10^{-6} & -10 \\ -1 \times 10^{-6} & -10 \times 10^{-3} \end{bmatrix}^2 = \begin{bmatrix} 10.25 \times 10^{-6} & 95 \times 10^{-3} \\ 9.5 \times 10^{-9} & 110 \times 10^{-6} \end{bmatrix}
\]

Consulting a table for terminated two-ports reveals that

\[
\frac{V_2}{V_g} = \frac{Z_L}{(A + CZ_g)Z_L + B + DZ_g} = \frac{10 \times 10^3}{(10.25 \times 10^{-6} + (9.5 \times 10^{-9})(500))(10 \times 10^3) + 95 \times 10^{-3} + (110 \times 10^{-6})(500)}
\]

\[
= 33.3 \times 10^3.
\]

Alternatively, we can represent the 500 \( \Omega \) generator resistance and the 10K with their \( ABCD \) matrices so that the overall \( ABCD \) matrix is

\[
\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 500 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 500 \times 10^{-6} & -10 \\ -1 \times 10^{-6} & -10 \times 10^{-3} \end{bmatrix} \begin{bmatrix} 500 \times 10^{-6} & -10 \\ -1 \times 10^{-6} & -10 \times 10^{-3} \end{bmatrix} \begin{bmatrix} 1 \ 0 \\ 100 \times 10^{-6} & 1 \end{bmatrix}.
\]

\[
= \begin{bmatrix} 30 \times 10^{-6} & 150 \times 10^{-3} \\ 20.5 \times 10^{-9} & 110 \times 10^{-6} \end{bmatrix}
\]

The desired voltage gain is \( 1/A = 1/(30 \times 10^{-6}) = 33.3 \times 10^3. \)
Question 4 The $h$-parameters for the two port is $h_{11} = 1K, h_{12} = 3 \times 10^{-3}, h_{21} = 100$ and $h_{22} = 50 \times 10^{-6}$ S. The input is a voltage source with $v_g = 10\angle 0^\circ$ mV and the voltage source has internal impedance of $Z_g = 500$ $\Omega$. Determine $v_2$ if $Z_L = 2K$. (20 points)

Note: you are strongly encouraged to use computer software such as Matlab to work this problem.

Solution For the calculations below, we use formulas from a table for terminated two-ports.

$$Z_{in} = \frac{v_1}{i_1} = h_{11} - \frac{h_{12}h_{21}Z_L}{1 + h_{22}Z_L} = 1 \times 10^3 - \frac{(3 \times 10^{-3})(100)(2 \times 10^3)}{1 + (50 \times 10^{-6})(2 \times 10^3)} = 454.54 \Omega$$

$Z_g$ and $Z_{in}$ form a voltage divider so that

$$v_1 = v_g \frac{Z_{in}}{Z_{in} + 500} = 4.76\angle 0^\circ \text{ mV}$$

$$\Delta h = h_{11}h_{22} - h_{12}h_{21} = (1,000)(50 \times 10^{-6}) - (3 \times 10^{-3})(100) = -0.25$$

$$v_2 = v_1 \left( \frac{-h_{21}Z_L}{\Delta hZ_L + h_{11}} \right) = (4.76\angle 0^\circ \text{ mV}) \left( \frac{-(100)(2 \times 10^3)}{(-0.25)(2 \times 10^3) + 1 \times 10^3} \right) = -1.904 \text{ V}$$

Alternatively, we can calculate $v_2$ directly:

$$v_2 = v_g \left( \frac{-h_{21}Z_L}{(h_{11} + Z_g)(1 + h_{22}) - h_{12}h_{21}Z_L} \right) = -1.904 \text{ V}$$

Below is a Matlab script for these calculations.

```matlab
% The h-parameters of the network.
h11 = 1000;   % h11
h12 = 0.003;  % h12
h21 = 100;    % h21
h22 = 50e-6;  % h22
h = [h11 h12; h21 h22];  % h matrix

% The generator voltage and internal impedance.
Zg = 500;   % Internal impedance
Vg = 10e-3; % Generator voltage
ZL = 2e3;   % Load resistance

% Calculations follow.
Zin = h11 - (h12*h21*ZL)/(1+h22*ZL)
V1 = Vg*(Zin)/(Zin+Zg)
V2 = V1*(-h21*ZL)/(det(h)*ZL+h11)
V2 = Vg*(-h21*ZL/((h11+Zg)*(1+h22*ZL)-h12*h21*ZL))
```