Radio Frequency Electronics

Impedance Matching I

- Born in 1914 in Austria
- Actress in Europe
- Married an arms dealer, who was very controlling
- Fleed husband and eventually ended up in Hollywood
- Made ~ 20 movies
- Was quite famous in 1940’s and 1950’s
- Invented frequency-hopping communication to control torpedoes to prevent jamming by enemy
- Was belated recognized for her invention in in 1980s and 1990s
- Died in 2000
Example. Design an RLC resonant circuit to provide a BW of 10 MHz at a center frequency $f_0 = 100$ MHz. The source and the load resistances are both 1K. Inductors are available with $Q = 85$ and assume lossless capacitors.

\[ L_s = 70.22 \text{ nH}, \quad R_s = 0.519 \Omega, \quad C = 36.1 \text{ pF} \]

where $R_s$ is the series resistance of the inductor. As a check, note that the inductor will transform its $R_s = 0.519 \Omega$ to a resistance value $R_p = (85^2)(0.519) = 3.75K$. This combines with the two 1K resistances from the source and load to form a resistance $R = 3.75K\|1K\|1K = 441.2 \Omega$. Further, the network $Q$ is

\[
Q_T = \frac{R}{\omega_0 L} = \frac{441.2}{(2\pi)(100 \times 10^6)(70.22 \times 10^{-9})} = 10
\]
Some Applications of Capacitive Transformers
Some Applications of Capacitive Transformers

50 MHz to 300MHz VCO with a tuning range of 2:1.
Some Applications of Capacitive Transformers

Where is the inductor?
Impedance Matching Example

Below is a SPICE simulation screen capture. An L matching network consisting of a 477 nH inductor and 4.8 pF capacitor transforms a 1K load into a 100 Ω at 100 MHz.

The input impedance was calculated in SPICE by plotting $\frac{V(V_i)}{I(I_m)}$. $V_i$ is the voltage at the input of the matching network and $I_m$ is a voltage source set to 0 V. This is a technique for measuring currents, since some SPICE implementations don’t have an ammeter built in.

Note that at 100 MHz, the magnitude of the input impedance is 100 Ω and the phase is 0, which means it is purely resistive.
There are 8 possible ways of using an inductor and capacitor to match a load impedance $Z_L$ to a source impedance $Z_S$.

These are called L-sections because they resemble an inverted letter L.

From “RF Circuit Design: Theory and Applications”, Ludwig & Bretchko
L- Network Impedance Matching

One can follow a completely analytical approach to finding the components in for the L network. See example 8.1 in Ludwig & Bretchko. This can become very tedious.

Another approach is to use the Smith chart and it is often the preferred method.

Yet another approach is to use the quality factor (Q-factor) concept in our calculations. This is the approach in Bowick and which we will explore first.

Here are the big picture steps:

- We want 1K to appear as 100 Ω
- Add a shunt capacitor that has a reactance \(-jX_C\)
- Calculating the input impedance shows we have the correct R, but there is a reactance \(-jX'_C\) in series
- No problem, add an inductor to cancel out the capacitor's reactance
- At the operating frequency the 1K resistor appears as a 100 Ω resistor
Consider a capacitor in parallel with a 1K resistor. The frequency and capacitance of the capacitor is such that its reactance is \(-j333.3\) Ω.

The network’s input impedance is \(100 - j300\) Ω. The resistance is 100Ω, 10 \(\times\) smaller than the actual resistor value.

There are many other networks that have the same input impedance at the specific frequency. One such network is a series RC network as shown.

If we now add a series inductor with reactance \(+j300\) Ω it will cancel the \(-j300\) reactance.

This makes the input impedance appear as a resistor with value 100 Ω.

Thus, we can transform a 1K resistor to a 100 Ω resistor using a capacitor and inductor.
The quality of $Q$-factor for the series and parallel legs of the $L$ match network are

\[
Q_s = \frac{X_s}{R_s}
\]

$X_s$ = Reactance of series leg

$R_s$ = Resistance of series leg

\[
Q_p = \frac{R_p}{X_p}
\]

$X_p$ = Reactance of parallel leg

$R_p$ = Resistance of parallel leg

We design the L network so that

\[
Q_s = Q_p = \frac{\sqrt{\frac{R_p}{R_s}} - 1}{\sqrt{\frac{R_p}{R_s}}}
\]

$R_p$ and $R_s$ come from the design requirement. For example “transform a 1K resistor into a 100 $\Omega$ resistor."
**Example.** Design a L-matching circuit to match a 100 Ω source to a 1K load at 100 MHz.

**Solution**

The design equations are

\[ Q_s = \frac{X_s}{R_s} = Q_p = \frac{R_p}{X_p} = \frac{R_p}{R_s} - 1 \]

Several of the L-networks will work. For this design, we choose a low pass configuration.

The Qs come from the design requirements:

\[ Q_s = Q_p = \frac{R_p}{R_s} - 1 = \frac{1,000}{10} - 1 = 3 \]

\[ Q_s = \frac{X_s}{R_s}, \quad X_s = 3R_g = 300 \Omega \]

\[ L = \frac{300}{(2\pi)(100 \times 10^6)} = 477 \text{ nH} \]

\[ Q_p = \frac{R_p}{X_p}, \quad X_p = \frac{R_p}{3} = \frac{1,000}{3} = 333.3 \Omega \]

\[ C = \frac{1}{(2\pi)(100 \times 10^6)(333.3)} = 4.8 \text{ pF} \]
Example. Design a L-matching circuit to match a 100 Ω source to a 1K load at 100 MHz.

Solution. The design equations are

\[ Q_s = \frac{X_s}{R_s} = Q_p = \frac{R_p}{X_p} = \frac{\sqrt{R_p/R_s} - 1}{R_s} \]

Same requirements as before, but now let’s choose a high-pass configuration.

The Qs come from the design requirements:

\[ Q_s = Q_p = \sqrt{\frac{R_p}{R_s} - 1} = \sqrt{\frac{1,000}{10} - 1} = 3 \]

\[ Q_s = \frac{X_s}{R_s} = 3R_g = 300 \Omega \]

\[ X_s = 3R_g = 300 \Omega \]

\[ C = \frac{1}{(2\pi)(100 \times 10^6)(300)} = 5.3 \text{ pF} \]

\[ Q_p = \frac{R_p}{X_p} \]

\[ X_p = \frac{R_p}{3} = \frac{1,000}{3} = 333.3 \Omega \]

\[ L = \frac{333.3}{(2\pi)(100 \times 10^6)} = 531 \text{ nH} \]
When have two designs that should work. Which one should we use?

One consideration may be whether we want to block dc or not. If we want to block dc, we would pick the high-pass version.

Another consideration may be the availability components. For example, in same matching problems the inductor/capacitor value may be inconvenient in one configuration, while the in the other configuration the values are no problematic.
Note that the series element connects to the side with the low resistance and the parallel element goes with the low resistance.

Remember it this way. We want to make the apparent $R_L$ smaller, so we put the reactive element in parallel with it.

Note that the series element connects to the side with the low resistance and the parallel element goes with the low resistance.

Remember it this way. We want to make the apparent $R_L$ larger, so we put the reactive element in series with it.
In the previous examples both the generator/source were purely resistive (100 Ω and 1K) respectively. Imagine the source and/or the load is complex as shown in below.

One technique for dealing with this is to absorb these elements into the matching network. First, ignore the elements and perform the matching as before.

Next we take out part of the matching network and move them to the terminations.

One can think of this as the generator supplying part of the reactance needed for matching.
Complex Sources and Loads

Another method is to add reactive elements that will resonate with and cancel them out.

This has a reactance of \(j(2\pi)(100 \times 10^6)(200 \times 10^{-9})\) or \(j(125.7)\ \Omega\) at 100 MHz.

A 12.67 pF capacitor will have a reactance \(-j(125.7)\ \Omega\) at 100 MHz.

We can add a 12.67 pF in series to cancel the 200 nH inductor’s effect at 100 MHz out.

Similarly, we can add a 1.27 μH inductor in parallel to cancel the 2 pF capacitor at the load.
That's All Folks