Radio Frequency Electronics

Active Components III

Samuel Morse

- Born in 1791 in Massachusetts
- Fairly accomplished painter
- After witnessing various electrical experiments, got intrigued by electricity
- Designed the first single-wire telegraph
- Invented the concept “relay” what we now call repeaters
- Created Morse Code (digital communications?)
- Held several patents related to the telegraph
- Dies in 1872
Junction & Diffusion Capacitance

Junction capacitance

\[ C_j = \frac{C_{j0}}{\left(1 + \frac{V_R}{V_{bi}}\right)^{m_{jc}}} \]

Diffusion capacitance associated with current flowing through the base-emitter junction

\[ C_d \approx \frac{I_C \tau_T}{V_T} = g_m \tau_T \]
The basic hybrid-\( \pi \) model for a BJT becomes more and more inadequate as the operating frequency increases.
We can view these elements as parasites that surround the basic BJT. There are called *parasitic* elements.
SPICE was not designed for RF work per se and has serious limitation in some areas.

However, with proper modeling, one can do quite good RF simulations with SPICE.

SPICE use more complex models than hybrid-π.

Some SPICE parameters match up with hybrid-π parameters, while other don’t.
\[ I_C = I_S e^{v_{BE}/V_T} \]

\[ C_b = \frac{I_C \tau_T}{V_T} = g_m \tau_T \]

\[ C_{\pi} = C_b + C_{je} \quad C_{\pi} \approx C_b = \frac{I_C \tau_T}{V_T} \]
\[ C_\mu = \frac{C_{JC}}{1 + \left( \frac{V_{CB}}{V_{JC}} \right)^{MJC}} \]

\[ I_C = IS e^{V_{BE}/V_T} \]

\[ C_b = \frac{I_C \tau_T}{V_T} = g_m \tau_T \]

\[ C_\pi = C_b + C_{je} \quad C_\pi \approx C_b = \frac{I_C \tau_T}{V_T} \]

\[ C_{je} = \frac{C_{JE}}{1 + \left( \frac{V_{EB}}{V_{JE}} \right)^{MJE}} \]

\[ r_{\mu} \]

\[ r_{\pi} \]

\[ r_{\text{ex}} \]

\[ \sim 1 \text{ to } 2 \Omega \]
Depending on the situation, we can ignore some of the parasitic elements.

In RF work we can seldom ignore highlighted elements.

\( C_\pi \) is normally \( \gg \) \( C_\mu \)

However, because of the feedback from \( C' \) to \( B' \) the effect of \( C_\mu \) can be much bigger than that of \( C_\pi \)

Both \( C_\pi \) and \( C_\mu \) are functions of Q-point
Short Circuit Gain

Ignore effect of $R_1$ and $R_2$

Coupling capacitor $\rightarrow \infty$

$R_L \rightarrow 0$

Current amplifier
Short-Circuit Current Gain: BJT Frequency Response

KCL at input

\[-I_b + \frac{V_\pi}{r_\pi} + \frac{V_\pi}{1/sC_\pi} + \frac{V_\pi}{1/sC_\mu} = 0\]

KCL at output

\[g_mV_\pi - I_C - \frac{V_\pi}{1/sC_\mu} = 0\]

\[I_C = V_\pi(g_m - sC_\mu)\]

\[A_i = \frac{I_c}{I_b} = h_{fe} = \frac{g_m - sC_\mu}{\frac{1}{r_\pi} + s(C_\pi + C_\mu)}\]

\[h_{fe} \approx \frac{g_m}{\frac{1}{r_\pi} + s(C_\pi + C_\mu)} = \frac{g_m r_\pi}{1 + s(C_\pi + C_\mu)r_\pi}\]

With typical values for \(C_\mu\) and \(g_m\)
Recall that at dc we used $r_\pi = \beta / g_m$, $\beta = g_m r_\pi$

$$f_\beta = \frac{1}{2\pi r_\pi (C_\pi + C_\mu)}$$  \hspace{1cm}  f_T = \beta_0 f_\beta$$

Beta cutoff frequency

Transition frequency
### Short-Circuit Current Gain: BJT Frequency Response

#### PN2222, PN2222A

**ELECTRICAL CHARACTERISTICS** *(T<sub>A</sub> = 25°C unless otherwise noted)* (Continued)

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Symbol</th>
<th>Min</th>
<th>Max</th>
<th>Unit</th>
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<tbody>
<tr>
<td><strong>SMALL–SIGNAL CHARACTERISTICS</strong></td>
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<td>Current–Gain – Bandwidth Product (Note 2.)</td>
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<tr>
<td><em>(I&lt;sub&gt;C&lt;/sub&gt; = 20 mAdc, V&lt;sub&gt;CE&lt;/sub&gt; = 20 Vdc, f = 100 MHz)</em></td>
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<td>Output Capacitance</td>
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<td><em>(V&lt;sub&gt;CB&lt;/sub&gt; = 10 Vdc, I&lt;sub&gt;E&lt;/sub&gt; = 0, f = 1.0 MHz)</em></td>
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| | f<sub>T</sub> | 250 | – | MHz |
| | f<sub>T</sub> | 300 | – | MHz |
| C<sub>obo</sub> | – | 8.0 | pF |
Miller Effect and Miller Capacitance

Small (~ 1 pF), but can have significant effect on frequency response
Physical Origin of Miller Effect

Inverting amplifier

Voltage gain from B to C (i.e., across $C_\mu$?)

$$C_M = C_\mu \left(1 - \text{voltage gain across } C_\mu \right)$$

$$C_M = C_\mu \left[1 + g_m \left(\frac{R_C}{R_L}\right) \right]$$

Answer = $-g_m R_L$


Inherent Resistances and Capacitances in n-Channel MOSFET

\[ C_{gs} \approx C_{gd} \approx \frac{1}{2} W L C_{ox} \]
Equivalent Circuit for n-Channel Common Source MOSFET
Unity-gain bandwidth is defined as the frequency where the magnitude of the short circuit current gain goes to 1.

\[ I_i = \frac{V_{gs}}{1/j\omega C_{gs}} + \frac{V_{gs}}{1/j\omega C_{gd}} \]

\[ I_d = g_m V_{gs} - \frac{V_{gs}}{1/j\omega C_{gd}} \]

\[ A_i = \frac{I_d}{I_i} = \frac{g_m - j\omega C_{gd}}{j\omega(C_{gs} + C_{gd})} \approx \frac{g_m}{j\omega(C_{gs} + C_{gd})} \]

\[ f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})} \]

\[ f_T = \frac{g_m}{2\pi(C_{\pi} + C_{\mu})} \]

Similar to BJT

KCL at input node

KCL at output node

Set to 1

Unity-Gain Bandwidth
MOSFET Miller Capacitance

Inverting amplifier

Voltage gain from G to D (i.e., across $C_{gd}$)

$$C_M = C_{gd} [1 + g_m R_L]$$

Answer = $-g_m R_L$
Base Spreading Resistance

$E_B$ Junction is forward-biased

Collector $p$

$C_B$ Junction is reverse-biased

Emitter

$p^{++}$

$n$ Base

$\mathcal{P}_{++}$
Base Spreading Resistance

$EB$ Junction is forward-biased

This resistance can often be ignored

$CB$ Junction is reverse-biased

Collector $p$

Emitter

$\text{ Junction is reverse-biased}$

$\text{ Junction is forward-biased}$

$\text{Collector } p$

$\text{Emitter}$

$\text{Junction is reverse-biased}$

$\text{Junction is forward-biased}$

$\text{Collector } p$

$\text{Emitter}$

$\text{Junction is reverse-biased}$

$\text{Junction is forward-biased}$
Base Spreading Resistance

- **EB Junction is forward-biased**
- **CB Junction is reverse-biased**

This resistance is the base-spreading resistance $r_b$, large, and can often not be ignored.

This resistance can often be ignored.

Collector $p$

Emitter

This resistance can often be ignored.
Base Spreading Resistance

Because of the base-spreading resistance, most of the hole injection occurs at the corners.

Problematic with high-power and high frequency devices.

$EB$ Junction is forward-biased

$CB$ Junction is reverse-biased
Emitter Crowding in $pnp$ BJT

$EB$ Junction is forward-biased

$CB$ Junction is reverse-biased

Emitter crowding at edges

Problematic with high-power and high frequency devices.
Emitter Crowding in $npn$ BTT

(a) Base $\rightarrow$ Emitter $\rightarrow$ Collector

(b) $I_B/2$ from Base to Collector

Collector current
Solution to Emitter Crowding

Since crowding occurs at edge, create base- and emitter connections with many edges
CE Amplifier (CS is Similar)

High-gain because of $C_E$

Inverting amplifier

Use time constant technique:

$$f_H = \frac{1}{2\pi \tau_p}$$

$$f_H = \frac{1}{2\pi [r_{\pi} \parallel R_B \parallel R_S]C_{eq}}$$
SPICE Results for Common Emitter
Notice where the input signal goes
Remember, it is $v_{BE}$ that controls collector current in BJTs

$$I_C = I_S e^{\frac{V_{BE}}{V_T}}$$
Designing a Common-Base Amplifier

**Design a CE amplifier**

- \( R_1, R_2, I_C, R_E, R_C \) are determined to achieve a desired operating point

**Ground the base**

- Feed signal into emitter

**\( A_v = -g_m R_C \)**

**\( A_v = - (40I_C) R_C = ?? \)**

- \( R_i = \frac{r\pi}{\beta} = \frac{1}{g_m} = ?? \)
This is not an inverting amplifier

Thus, Miller no multiplication effect.
These are NOT inverting amplifiers.

Thus, Miller no multiplication effect.
CB Amplifier

Equivalent input circuit

$$Z_i = R_S + \frac{r_\pi}{1 + \beta} \left( R_E \| R_S \right) C_\pi$$

$$v_i = -V_\pi$$

$$v_e = -V_\pi$$

Equivalent output circuit

$$f_{H\mu} = \frac{1}{2\pi (R_C \| R_L) C_\mu}$$

Either one could determine bandwidth (normally $C_\mu$)

Regardless, higher bandwidth than CE

$$f_{H\pi} = \frac{1}{2\pi \left( \frac{r_\pi}{1 + \beta} \left( R_E \| R_S \right) \right) C_\pi}$$
CE is an inverting amplifier => Miller effect present

CE voltage gain ~ 1 => low Miller effect
Cascode Circuit

\[ f_{H\pi} = \frac{1}{2\pi \left( R_S \parallel R_{B1} \parallel r_{\pi1} \right) \left( C_{\pi1} + C_{M1} \right)} \]

Either one could determine bandwidth (normally \( C_{\mu} \))

\[ f_{H\mu} = \frac{1}{2\pi \left( R_C \parallel R_L \right) C_{\mu2}} \]

Wide bandwidth
Emitter-Follower Circuit (Source-Follower is Similar)
Wide bandwidth
SPICE Results for Emitter Follower

$|A_V|$

$C_\pi$ only

$C_\pi$ and $C_\mu$ only

$C_L = 150$ pF

$f(\text{Hz})$
Single-Tuned Amplifier

Tuned amplifier using a depletion-mode MOSFET

The equivalent ac circuit

Circuit for bias calculations

The small-signal model
The small-signal model

KCL @ output

\[(v_o(s) - v_i(s))sC_{GD} + g_m v + v_o(s) \left( \frac{1}{r_o} + \frac{1}{sL} + sC + \frac{1}{R_D||R_3} \right) = 0 \]

Let \( R_p = 1/G_p \) where \( G_p = 1/r_o + 1/R_D + 1/R_3 \) (i.e., the parallel combination of the resistances at the output. Solving for the voltage transfer function \( v_o(s)/v_i(s) \) yields

\[ A_v(s) = \frac{v_o(s)}{v_i(s)} = (sC_{GD} - g_m)R_p \frac{s}{s^2 + \frac{R_p(C + C_{GD})}{s} + \frac{1}{L(C + C_{GD})}} \]

Neglecting the right-half-plane zero ("sC_{GD}” in \( sC_{GD} - g_m \)) then

\[ A_v(s) \approx A_{mid} \frac{s \frac{\omega}{Q}}{s^2 + s \frac{\omega}{Q} + \omega_0^2}, \quad \omega_0 = \frac{1}{\sqrt{L(C + C_{GD})}}, \quad Q = \omega_0 R_p (C + C_{GD}), \quad A_{mid} = -g_m R_p \]

Further, \( Q = R_p/\omega L \) and \( BW = \omega_0/Q \)
\[ A_v(s) \approx A_{mid} \frac{s \frac{\omega}{Q}}{s^2 + s \frac{\omega}{Q} + \omega_0^2}, \]

\[ \omega_0 = \frac{1}{\sqrt{L(C + C_{GD})}}, \]

\[ Q = \omega_0 R_P (C + C_{GD}) \]

\[ A_{mid} = -g_m R_P \]

Assume \( \lambda = 0.02 \, V^{-1} \), \( I_D = 3.2 \, mA \), \( C_{GD} = 20 \, pF \)

\[ g_m = 2\sqrt{K_n I_D} = 2\sqrt{(2.5 \times 10^{-3})(3.2 \times 10^{-3})} = 5.66 \, mA/V^2 \]

\[ r_o = 1/\lambda I_D = 1/(0.02)(3.2 \times 10^{-3}) = 15.6K \]

\[ R_p = r_o || 100K || 100K = 11.9K \]

\[ C + C_{GD} = 100 + 20 = 120 \, pF \]

\[ \omega_0 = 1/\sqrt{L(C + C_{GD})} = 1/\sqrt{(10 \times 10^{-6})(120 \times 10^{-12})} = 28.6 \times 10^6 \, \text{rad/s} \]

\[ f_0 = \omega_0 / 2\pi = 4.59 \, \text{MHz} \]

\[ A_{mid} = -g_m R_p = -(5.66 \times 10^{-3})(11.9 \times 10^3) = -67 \]

\[ Q = \omega_0 R_P (C + C_{GD}) = -(28.6 \times 10^6)(11.9 \times 10^3)(120 \times 10^{-12}) = 40.8 \]

\[ BW = f_0 / Q = 4.59 \times 10^6 / 40.8 = 112 \, \text{kHz} \]
**Small-Signal Models**

Simple small signal model for BJT

\[
g_m = \frac{I_{CQ}}{V_T} \approx 40 I_{CQ}
\]

\[
r_{\pi} = \frac{\beta V_T}{I_{CQ}}
\]

\[
g_m r_{\pi} = \beta
\]

\[
r_o = \frac{V_A}{I_{CQ}}
\]
Figure 7-38  Basic circuit topology of the Curtice Quadratic GaAs MESFET model employed for the transistor die.
That's All Folks