### Diodes

\[ I_D = I_S \left[ \frac{V_D}{e^{\frac{V_D}{V_T}} - 1} \right] \]

\[ r_d = \frac{1}{g_d} = \frac{V_T}{I_{DQ}} \]

\[ C_D = \frac{I_D \tau_T}{V_T} = g_d \tau_T \]

\[ C_j = \frac{C_{j0}}{(1 + V_R/V_{bi})^m} \]

### BJTs

<table>
<thead>
<tr>
<th>Collector-base junction capacitance</th>
<th>[ C_j = \frac{C_{j0}}{(1 + \frac{V_R}{V_{bi}})^{mJC}} ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base-emitter junction capacitance</td>
<td>[ C_{\pi} = \frac{g_m}{\omega_T} - C_{\mu} \approx \frac{g_m}{\omega_T}, \quad C_{\pi} \approx g_m \tau_T, \quad C_{BE} = g_m T_F ]</td>
</tr>
<tr>
<td>Beta-cutoff frequency</td>
<td>[ f_\beta = \frac{1}{2\pi r_\pi (C_{\pi} + C_{\mu})} ]</td>
</tr>
<tr>
<td>Gain-bandwidth product</td>
<td>[ f_T = \beta_0 f_\beta, \quad \omega_T = \frac{g_m}{C_{\pi} + C_{\mu}} ]</td>
</tr>
</tbody>
</table>

### h-Parameters

\[ v_1 = h_{11} i_1 + h_{12} v_2 \]
\[ i_2 = h_{21} i_1 + h_{22} v_2 \]
\[ h_{11} = \left. \frac{v_1}{i_1} \right|_{v_2=0} \quad h_{12} = \left. \frac{v_1}{v_2} \right|_{i_1=0} \]
\[ h_{21} = \left. \frac{i_2}{i_1} \right|_{v_2=0} \quad h_{22} = \left. \frac{i_2}{v_2} \right|_{i_1=0} \]
BJT CE Version

\[
\begin{align*}
V_{be} &= h_{ie}i_b + h_{re}V_{ce} \\
i_c &= h_f i_b + h_{oe}V_{ce}
\end{align*}
\]

\[
h_{ie} = r_b + r || \mu \\
h_{fe} = \beta \\
h_{re} = \frac{r \| \mu}{r} \\
h_{oe} = \frac{1 + \beta + 1}{r \| \mu}
\]

\textbf{y-Parameters}

\[
\begin{align*}
i_1 &= y_{11}v_1 + y_{12}v_2 \\
i_2 &= y_{21}v_1 + y_{22}v_2 \\
y_{11} &= \left. \frac{\Delta i_1}{\Delta v_1} \right|_{v_2 = 0} \\
y_{12} &= \left. \frac{\Delta i_1}{\Delta v_2} \right|_{v_1 = 0} \\
y_{21} &= \left. \frac{\Delta i_2}{\Delta v_1} \right|_{v_2 = 0} \\
y_{22} &= \left. \frac{\Delta i_2}{\Delta v_2} \right|_{v_1 = 0}
\end{align*}
\]
RL, RC Series $\leftrightarrow$ Parallel Transformations

\[ Q_s = \frac{|X_s|}{R_s}, \quad Q_p = \frac{R_p}{|X_p|} \]

(a) \[ R_p = R_s (1 + Q_s^2) \]
\[ L_p = L_s \left( \frac{1 + Q_s^2}{Q_s^2} \right) \]
\[ Q_s = \frac{\omega L_s R_s}{Q_s^2} \]

(b) \[ R_s = \frac{R_p}{1 + Q_p^2} \]
\[ L_s = L_p \left( \frac{Q_p^2}{1 + Q_p^2} \right) \]
\[ Q_p = \frac{R_p}{\omega L_p} \]

(c) \[ R_p = R_s (1 + Q_s^2) \]
\[ C_p = C_s \left( \frac{Q_s^2}{1 + Q_s^2} \right) \]
\[ Q_s = \frac{1}{\omega R_s C_s} \]

(d) \[ R_s = \frac{R_p}{1 + Q_p^2} \]
\[ C_s = C_p \left( \frac{1 + Q_p^2}{Q_p^2} \right) \]
\[ Q_p = \omega R_p C_p \]
Fourier Series

\[ x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(2\pi f t n) - \sum_{n=1}^{\infty} b_n \sin(2\pi f t n) \]

\[ a_0 = \frac{1}{T} \int_{-T/2}^{T/2} x(t) \, dt \]

\[ b_n = -\frac{2}{T} \int_{-T/2}^{T/2} x(t) \sin\left(\frac{2\pi t n}{T}\right) \, dt \]

\[ a_n = \frac{2}{T} \int_{-T/2}^{T/2} x(t) \cos\left(\frac{2\pi t n}{T}\right) \, dt \]

a. Pulse

\[ a_0 = A d \]

\[ a_n = \frac{2A}{n\pi} \sin(n\pi d) \]

\[ b_n = 0 \]

(d = 0.27 in this example)

b. Square

\[ a_0 = 0 \]

\[ a_n = \frac{2A}{n\pi} \sin\left(\frac{n\pi}{2}\right) \]

\[ b_n = 0 \]

(all even harmonics are zero)
Second-Order Transfer Function Summary

Standard second-order form (s-plane)

\[
H(s) = \frac{N(s)}{s^2 + 2\zeta \omega_0 s + \omega_0^2} = \frac{N(s)}{\left(\frac{s}{\omega_0}\right)^2 + \frac{2\zeta}{\omega_0} s + 1} = \frac{\omega_0^2}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}
\]

Where \( Q = 1/2\zeta \), and \( B = \omega_0/Q \), and \( \omega_0 \) is the undamped natural frequency (rad/s), and \( \zeta \) (Zeta) is the damping ratio, and \( Q \) is the quality factor, and \( B \) is the 3-dB bandwidth in (rad/s)

Case \( \zeta > 1 \) (Overdamped): The poles are real and negative. The natural response consists of two decaying exponentials.

Case \( 0 < \zeta < 1 \) (Underdamped): The poles are complex conjugate

\[
p_{1,2} = -\zeta \omega_0 \pm j\omega_0\sqrt{1 - \zeta^2}
\]

The natural response is a damped sinusoid \( (A \) is the residue at the upper pole):

\[
x_0(t) = 2|A|e^{-\zeta \omega_0 t} \cos \left(\omega_0\sqrt{1 - \zeta^2} t + \angle A\right)
\]

Percentage overshoot (PO) with step response

\[
PO = 100e^{-\pi\zeta / \sqrt{1 - \zeta^2}}
\]

Case \( \zeta = 0 \) (No damping): The poles are complex conjugate: \( p_{1,2} = \pm j\omega_0 \). The natural response is a sustained sinusoid with frequency \( \omega_0 \)

Case \( \zeta < 0 \) (Unstable): The poles lie in the right-hand plane causing a diverging response.
\[
\begin{align*}
\cos(a) \cos(b) &= \frac{1}{2} \left( \cos(a + b) + \cos(a - b) \right) \\
\sin(a) \sin(b) &= \frac{1}{2} \left( \cos(a - b) - \cos(a + b) \right) \\
\sin(a) \cos(b) &= \frac{1}{2} \left( \sin(a + b) + \sin(a - b) \right) \\
\cos(a) \sin(b) &= \frac{1}{2} \left( \sin(a + b) - \sin(a - b) \right)
\end{align*}
\]